Information retrieval

Language models

Corso di Laurea Magistrale in Informatica

Università di Roma Tor Vergata

Prof. Giorgio Gambosi

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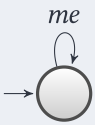
Using language models (LMs) for IR

- We view the document in terms of as a generative model that generates the query
- What we need to do:
  - Define the precise generative model we want to use
  - Estimate parameters (different parameters for each document's model)
  - Smooth to avoid zeros
  - Apply to query and find document most likely to have generated the query
  - Present most likely document(s) to user

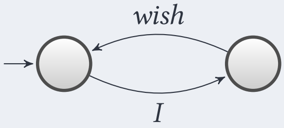
What is a language model?

- Assume we are reading (or generating) a document d term by term
- We can view a language model M_d for d as a way to determine the next term which will be read (generated)

We can view the language model as a finite state automaton, where the transitions between states are associated to terms



me me me me …



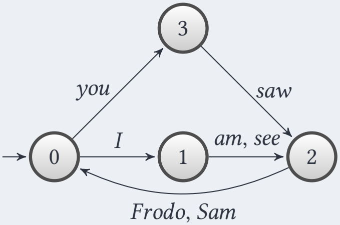
I wish I wish …

Cannot generate: “I I”, “ wish wish wish” or “wish I wish”: history counts

Probabilistic language models

Each document was generated by a different automaton like this, except that these automata are probabilistic.

- For each node, a probability distribution is defined on all transitions
- A document corresponds to (is generated as) a sequence of random sample on such distributions

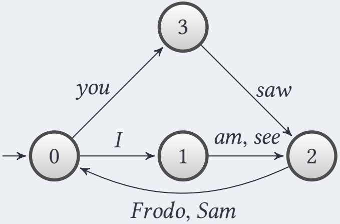


$P(\text{term}|\text{state})$

t	$p(t\|0)$	$p(t\|1)$	$p(t\|2)$	$p(t\|3)$
I	0.4			
you	0.4			
am		0.5		
see		0.5		
saw				1
Frodo			0.8	
Sam			0.2	
STOP	0.2			

- This is a probabilistic finite-state automaton and the transition distribution for its states 0, 1, 2, 3.
- STOP is not a word, but a special symbol indicating that the automaton stops.

A probabilistic language model



$P(\text{term}|\text{state})$

t	$p(t\|0)$	$p(t\|1)$	$p(t\|2)$	$p(t\|3)$
I	0.4			
you	0.4			
am		0.5		
see		0.5		
saw				1
Frodo			0.8	
Sam			0.2	
STOP	0.2			

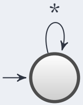
- possible sequence generated:

w	I	am	Frodo	you	saw	Sam	STOP
$P(t\|s)$	0.4	0.5	0.8	0.4	1	0.2	0.2

Total sequence probability = 0.00256

A probabilistic unigram language model

A simple version of language models, that we will consider here, is provided by the case when there is a unique state, hence $p(t|s) = p(t)$ for each term



$P(\text{term})$

t	$p(t)$
I	0.1
you	0.05
am	0.1
see	0.15
saw	0.1
Frodo	0.3
Sam	0.1
STOP	0.1

- possible sequence generated:

w	Frodo	I	Sam	Sam	saw	see	Frodo	STOP
$P(t\|s)$	0.3	0.1	0.1	0.1	0.15	0.2	0.3	0.1

Total sequence probability $= 2.7 \cdot 10^{-7}$

A different language model for each document

M_{d_1}: language model of d_1

t	$p(t)$	t	$p(t)$
STOP	.1	I	.15
you	.2	am	.05
see	.05	saw	.05
Frodo	.2	Sam	.2

M_{d_1}: language model of d_1

t	$p(t)$	t	$p(t)$
STOP	.2	I	.1
you	.1	am	.05
see	.15	saw	.2
Frodo	.15	Sam	.05

query: Frodo saw Sam STOP

$$p(\text{query}|M_{d1}) = 0.2 \cdot 0.05 \cdot 0.2 \cdot 0.1 = 2 \cdot 10^{-4}$$
$$p(\text{query}|M_{d2}) = 0.15 \cdot 0.2 \cdot 0.05 \cdot 0.2 = 3 \cdot 10^{-4}$$

$p(\text{query}|M_{d1}) < p(\text{query}|M_{d2})$: thus, document d_2 is “more relevant” to the query “Frodo saw Sam STOP ” than d_1 is.

Using language models in IR

- Each document is treated as (the basis for) a language model.
- Given a query q:
  - We wish to rank documents by

$$p(d|q) = \frac{p(q|d)p(d)}{p(q)}$$

  - $p(q)$ is the same for all documents, so ignore
  - $p(d)$ is the prior – often treated as the same for all d
    - But we could give a higher prior to documents which are relevant wrt some other measure, e.g., those with high PageRank.
  - $p(q|d)$ is the probability of q given d
  - For uniform prior: ranking documents according to $p(q|d)$ and $p(d|q)$ is equivalent.

Where we are

We may see $p(q|d)$ as the probability that the document the user had in mind when she was formulating the query was in fact this one.

- In the LM approach to IR, we attempt to model the query generation process.
- Then we rank documents by the probability that a query would be observed as a random sample from their respective document models (probability distributions)
- That is, we rank according to $P(q|d)$.
- In general, a document model structure (type of probability distribution) is assumed and its parameters values are derived, for each document, from its content

How to compute $P(q|d)$

- We make the Naive Bayes conditional independence assumption:

$$p(q|M_d) = p(< w_1, \dots, w_{|q|} > |M_d) = \prod_{i=1}^{|q|} p(w_k|M_d)$$

$|q|$: length of q; w_k: token t occurring at position k in q

How to compute $P(q|d)$

Where do the parameters $p(w_k|M_d)$ come from?

- Likelihood: this is the probability of data given a model, in this case $p(q|M_d)$
  - For fixed data, this provides a measure associated to each model instance (parameter values): the probability that such data are generated in the probabilistic framework defined by the model instance (for example, probability distribution)
  - This can be seen as “how much” a model instance explains the given data

How to compute $P(q|d)$

An hypothesis on the model structure (and the generation process) must be assumed.

- hypothesis: all terms have an associated probability to be the next word generated; this probability is independent from previous occurrences
- the probability of observing k occurrences of term t in the query q is given by the binomial distribution

$$p(\mathrm{tf}_{t,q} = k) = \frac{|q|!}{k!(|q|-k)!} p^k (1-p)^{|q|-k}$$

- the probability of observing $k_1, k_2, \dots, k_m$ occurrences of all terms $t_1, \dots, t_m$ in the query q is given by the multinomial distribution

$$p(\mathrm{tf}_{t_i,q} = k_i, i = 1, \dots, m) = \frac{|q|!}{\prod_{i=1}^m k_i!} \prod_{i=1}^m p_i^{k_i}$$

How to compute $P(q|d)$

- That is, for each document d,

$$p(\mathrm{tf}_{t_i,q} = k_i, i = 1, \dots, m|M_d) \approx \prod_{t \in q} p(t|M_d)^{\mathrm{tf}_{t,q}}$$

since the multiplying factor

$$\frac{|q|!}{\prod_{i=1}^{m} \mathrm{tf}_{t_i,q}!}$$

is independent from the document

- here M_d is an m-dimensional array

$$M_d = [p_1, p_2, \dots p_m]$$

with $\sum_{i=1}^{m} p_i = 1$ and $p(t_i|M_d) = p_i$

Parameter estimation

- The probability of the term in the document model, estimated by maximum likelihood is

$$\hat{p}_i = \hat{p}(t_i|M_d) = \frac{\mathrm{tf}_{t_i,d}}{|d|}$$

- $|d|$: length of d
- $\mathrm{tf}_{t_i,d}$: #occurrences of t_i in d

Different models

Different hypotheses on the distribution (generative process) provide different estimations.

- Multiple Poisson: we assume a dependancy exists between occurrences of a term. This is formalized by a Poisson distribution

$$p(\mathrm{tf}_{t,q} = k) = \frac{e^{-\lambda|q|}(\lambda|q|)^k}{k!}$$

  where $\lambda|q|$ is the expected number of occurrences of t in q
- for the whole query

$$p(\mathrm{tf}_{t_i,q} = k_i, i = 1, \dots, m|M_d) = \prod_{i=1}^{m} \frac{e^{-\lambda_i|q|}(\lambda_i|q|)^{k_i}}{k_i!}$$

- here M_d is an m-dimensional array

$$M_d = [\lambda_1, \lambda_2, \dots \lambda_m]$$

Smoothing in the multinomial model

- We have a problem with zeros: a single t with $p(t|M_d) = 0$ will make $p(q|M_d) = \prod p(t|M_d) = 0$
- We would give a single term "veto power".
- For example, for query [Frodo goes to mount Doom] a document about "Frodo Sam Doom" would have $p(q|M_d) = 0$
- We need to smooth the estimates to avoid zeros.

Laplace smoothing

- Key intuition: A nonoccurring term is possible (even though it didn't occur), so we don't want to assign 0 probability to it
- We may avoid the zero probability case by adding a constant value, such as 1, to the count $\mathrm{tf}_{t,d}$ in the maximum likelihood estimation: the numerator of the estimation ratio is now $\mathrm{tf}_{t,d} + 1$
- This eliminates the zero probability case, but makes the normalization wrong, that is $\sum_t \mathrm{tf}_{t,d} > |d|$. This can be avoided by summing M, the overall number of terms to $|d|$ at the denominator

$$\hat{p}(t|M_d) = \frac{\mathrm{tf}_{t,d} + 1}{|d| + M}$$

Smoothing through collection model

- We may estimate its probability of a term in a document model by looking at the whole collection
- Let us consider the collection model M_c the collection model: we may estimate
- The maximum likelihood estimate of the probability of the term in the whole collection is given by

$$\hat{p}(t|M_c) = \frac{\mathrm{cf}_t}{T}$$

  where cf_t is the number of occurrences of t in the collection and $T = \sum_t \mathrm{cf}_t$ is the total number of tokens in the collection.
- We will use (the estimate of) $\hat{p}(t|M_c)$ to "smooth" (the estimate of) $p(t|d)$ away from zero.

Jelinek-Mercer smoothing

The estimated probability of the term wrt the document is defined as a linear combination of the probability according to the document model and the probability according to the collection model

$$p(t|d) = \lambda p(t|M_d) + (1 - \lambda)p(t|M_c)$$

λ is a hyper-parameter which tunes the relevance of the document model wrt the collection model

- Mixtures of two distributions
- Correctly setting λ is very important for good performance

Jelinek-Mercer smoothing

Assuming the conditional independence of terms,

$$p(q|d) = \prod_{1 \leq k \leq |q|} \Big(\lambda p(t_k|M_d) + (1-\lambda) p(t_k|M_c) \Big)$$

- Basic idea: we model the case that the user has a document in mind and generates the query from this document.
-
  - High value of λ: "conjunctive-like" search – tends to retrieve documents containing all query words.
  - Low value of λ: more disjunctive, suitable for long queries

Example

Collection:

- d_1: "Frodo and Sam reached mount Doom with the help of Gollum"
- d_2: "Gollum was attracted by the One Ring"

Query q: "Gollum Ring"

- Use mixture model with $\lambda = 1/2$
- $|d_1| = 11$, $|d_2| = 7$, $T = 18$
- $P(q|d_1) = [(1/11 + 2/18)/2] \cdot [(0/11 + 1/18)/2] =\approx 0.0028$
- $P(q|d_2) = [(1/7 + 2/18)/2] \cdot [(1/7 + 1/18)/2] \approx 0.0125$
- Ranking: $d_2 > d_1$

Exercise: Compute ranking

- Collection: d_1 and d_2
- d_1: Frodo had a small sword and a coat
- d_2: The Shire was a small region in the west of Middle Earth
- Query q: west small
- Use mixture model with $\lambda = 1/2$
- $|d_1| = 8$, $|d_2| = 12$, $T = 20$

t	tf_{t,d_1}	tf_{t,d_2}	cf_t
west	0	1	1
small	1	1	2

- ...

Dirichlet smoothing

This is a bayesian approach: M_d is a random variable, with an associated distribution.

- a prior distribution $p(M_d)$ is defined from the collection model M_c: it is a distribution of probabilities of m-dimensional vectors (summing to 1) with expected value M_c and other parameters predefined
- by observing the document d content, a posterior distribution $p(M_d|d)$ is derived by applying Bayes rule

$$p(M_d|d) = \frac{p(d|M_d)p(M_d)}{p(d)}$$

- $p(d|M_d)$ is a multinomial distribution, by hypothesis
- $p(M_d)$ is chosen in such a way that $p(M_d)$ and $p(M_d|d)$ are of the same type (conjugate to the multinomial)
- typical choice, Dirichlet distribution

Dirichlet smoothing

- Dirichlet distribution

$$Dir(p_1, \dots, p_m | \alpha_1, \dots, \alpha_m) = \frac{\Gamma(\sum_{i=1}^{m})\alpha_i)}{\prod_{i=1}^{m} \Gamma(\alpha_i)} \prod_{i=1}^{m} p_i^{\alpha_i - 1}$$

- assume the prior distribution is defined as $\alpha_i = \mu \cdot p(t_i|M_c)$, where $p(t_i|M_c)$ is estimated as before
- under this hypothesis, it can be proved that the posterior distribution is then

$$\begin{aligned} p(M_d|d) &= p(p(t_1|M_d), \dots, p(t_m|M_d)|d, M_c, \mu) \\ &\simeq \prod_{i_1}^{m} p(t_i|M_d)^{\mathrm{tf}_{t,d} + \mu \cdot p(t_i|M_c) - 1} \end{aligned}$$

a Dirichlet with parameters $\alpha_i = \mathrm{tf}_{t_i,d} + \mu \cdot p(t_i|M_c)$

The resulting document model M_d is the expectation of the posterior distribution

- in general, in a Dirichlet distribution the expectation is the m-dimensional array with components

$$\frac{\alpha_i}{\sum_{k=1}^{m} \alpha_k}$$

- as a consequence, M_d is the m-dimensional array with components

$$\frac{\mathrm{tf}_{t_i,d} + \mu \cdot p(t_i|M_c)}{\sum_{k=1}^{m}(\mathrm{tf}_{t_k,d} + \mu \cdot p(t_k|M_c))} = \frac{\mathrm{tf}_{t_i,d} + \mu \cdot p(t_i|M_c)}{|d| + \mu}$$

Dirichlet smoothing

- Intuition: Before having seen any part of the document we start with the background distribution as our estimate.
- As we read the document and count terms we update the background distribution.
- The weighting factor μ determines how strong an effect the prior has.

Jelinek-Mercer or Dirichlet?

- Dirichlet performs better for keyword queries, Jelinek-Mercer performs better for verbose queries.
- Both models are sensitive to the smoothing parameters – you shouldn't use these models without parameter tuning.

Vector space vs BM25 vs LM

- BM25/LM: based on probability theory
- Vector space: based on similarity, a geometric/linear algebra notion
- Term frequency is directly used in all three models.
  - LMs: raw term frequency, BM25/Vector space: more complex
- Length normalization
  - Vector space: document vectors normalized
  - LMs: probabilities are inherently length normalized
  - BM25: tuning parameters for optimizing length normalization
- idf: BM25/vector space use it directly.
- LMs: Mixing term and collection frequencies has an effect similar to idf.
  - Terms rare in the general collection, but common in some documents will have a greater influence on the ranking.
- Collection frequency (LMs) vs. document frequency (BM25, vector space)