Text classification & Naive Bayes

Giorgio Gambosi

Course of Information Retrieval
CdLM in Computer Science
University of Rome Tor Vergata

Derived from slides produced by C. Manning and by H. Schütze

A text classification task: Email spam filtering

```
From: ``'' <takworlld@hotmail.com>
Subject: real estate is the only way... gem  oalvgkay

Anyone can buy real estate with no money down

Stop paying rent TODAY !

There is no need to spend hundreds or even thousands for similar courses

I am 22 years old and I have already purchased 6 properties using the
methods outlined in this truly INCREDIBLE ebook.

Change your life NOW !

=================================================
Click Below to order:
http://www.wholesaledaily.com/sales/nmd.htm
=================================================
```

How would you write a program that would automatically detect and delete this type of message?

Formal definition of TC: Training

Given:

- A document space $\mathbb{X}$
  - Documents are represented in this space – typically some type of high-dimensional space.
- A fixed set of classes $\mathbb{C} = \{c_1, c_2, \ldots, c_J\}$
  - The classes are human-defined for the needs of an application (e.g., spam vs. nonspam).
- A training set $\mathbb{D}$ of labeled documents. Each labeled document $\langle d, c \rangle \in \mathbb{X} \times \mathbb{C}$

Using a learning method or learning algorithm, we then wish to learn a classifier γ that maps documents to classes:

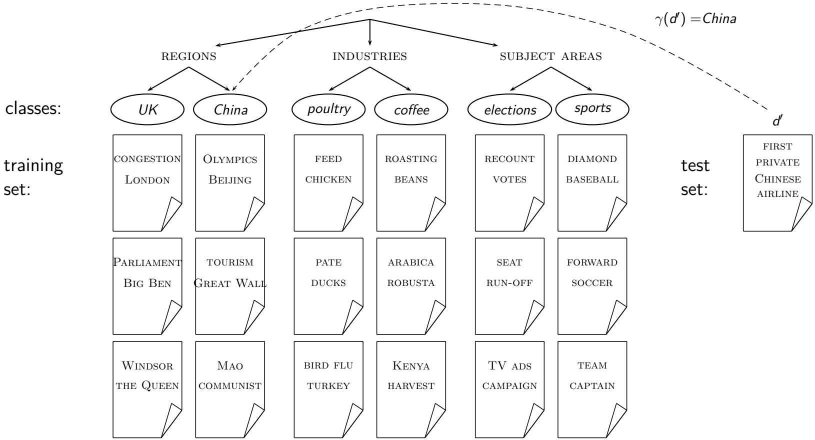
$$\gamma : \mathbb{X} \to \mathbb{C}$$

Formal definition of TC: Application/Testing

Given: a description $d \in \mathbb{X}$ of a document

Determine: $\gamma(d) \in \mathbb{C}$, that is, the class that is most appropriate for d

Topic classification



Exercise

- Find examples of uses of text classification in information retrieval

Examples of how search engines use classification

- Language identification (classes: English vs. French etc.)
- The automatic detection of spam pages (spam vs. nonspam)
- Sentiment detection: is a movie or product review positive or negative (positive vs. negative)
- Topic-specific or *vertical* search – restrict search to a "vertical" like "related to health" (relevant to vertical vs. not)

Classification methods: 1. Manual

- Manual classification was used by Yahoo in the beginning of the web. Also: ODP, PubMed
- Very accurate if job is done by experts
- Consistent when the problem size and team is small
- Scaling manual classification is difficult and expensive.
- $\rightarrow$ We need automatic methods for classification.

Classification methods: 2. Rule-based

- E.g., Google Alerts is rule-based classification.
- There are IDE-type development enviroments for writing very complex rules efficiently. (e.g., Verity)
- Often: Boolean combinations (as in Google Alerts)
- Accuracy is very high if a rule has been carefully refined over time by a subject expert.
- Building and maintaining rule-based classification systems is cumbersome and expensive.

A Verity topic (a complex classification rule)

comment line	`# Beginning of art topic definition`
top-level topic	`art ACCRUE`
topic definition modifiers	`/author = "fsmith"`
	`/date = "30-Dec-01"`
	`/annotation = "Topic created by fsmith"`
subtopictopic	`* 0.70 performing-arts ACCRUE`
evidencetopic	`** 0.50 WORD`
topic definition modifier	`/wordtext = ballet`
evidencetopic	`** 0.50 STEM`
topic definition modifier	`/wordtext = dance`
evidencetopic	`** 0.50 WORD`
topic definition modifier	`/wordtext = opera`
evidencetopic	`** 0.30 WORD`
topic definition modifier	`/wordtext = symphony`
subtopic	`* 0.70 visual-arts ACCRUE`
	`** 0.50 WORD`
	`/wordtext = painting`
	`** 0.50 WORD`
	`/wordtext = sculpture`
subtopic	`* 0.70 film ACCRUE`
	`** 0.50 STEM`
	`/wordtext = film`
subtopic	`** 0.50 motion-picture PHRAS`
	`*** 1.00 WORD`
	`/wordtext = motion`
	`*** 1.00 WORD`
	`/wordtext = picture`
	`** 0.50 STEM`
	`/wordtext = movie`
subtopic	`* 0.50 video ACCRUE`
	`** 0.50 STEM`
	`/wordtext = video`
	`** 0.50 STEM`
	`/wordtext = vcr`
	`# End of art topic`

Classification methods: 3. Statistical/Probabilistic

- This was our definition of the classification problem – text classification as a learning problem
- (i) Supervised learning of a the classification function γ and (ii) application of γ to classifying new documents
- We will look at two methods for doing this: Naive Bayes and SVMs
- No free lunch: requires hand-classified training data
- But this manual classification can be done by non-experts.

The Naive Bayes classifier

- The Naive Bayes classifier is a probabilistic classifier.
- We compute the probability of a document d being in a class c as follows:

$$P(c|d) \propto P(c) \prod_{1 \leq k \leq n_d} P(t_k|c)$$

- n_d is the length of the document. (number of tokens)
- $P(t_k|c)$ is the conditional probability of term t_k occurring in a document of class c
- $P(t_k|c)$ as a measure of how much evidence t_k contributes that c is the correct class.
- $P(c)$ is the prior probability of c.
- If a document's terms do not provide clear evidence for one class vs. another, we choose the c with highest $P(c)$.

Maximum a posteriori class

- Our goal in Naive Bayes classification is to find the "best" class.
- The best class is the most likely or maximum a posteriori (MAP) class c_{map}:

$$c_{\text{map}} = \arg\max_{c \in \mathbb{C}} \hat{P}(c|d) = \arg\max_{c \in \mathbb{C}} \ \hat{P}(c) \prod_{1 \le k \le n_d} \hat{P}(t_k|c)$$

Taking the log

- Multiplying lots of small probabilities can result in floating point underflow.
- Since $\log(xy) = \log(x) + \log(y)$, we can sum log probabilities instead of multiplying probabilities.
- Since log is a monotonic function, the class with the highest score does not change.
- So what we usually compute in practice is:

$$c_{\text{map}} = \arg\max_{c \in \mathbb{C}} [\log \hat{P}(c) + \sum_{1 \leq k \leq n_d} \log \hat{P}(t_k|c)]$$

Naive Bayes classifier

- Classification rule:

$$c_{\text{map}} = \arg\max_{c \in \mathbb{C}} \left[\log \hat{P}(c) + \sum_{1 \leq k \leq n_d} \log \hat{P}(t_k|c) \right]$$

- Simple interpretation:
  - Each conditional parameter $\log \hat{P}(t_k|c)$ is a weight that indicates how good an indicator t_k is for c.
  - The prior $\log \hat{P}(c)$ is a weight that indicates the relative frequency of c.
  - The sum of log prior and term weights is then a measure of how much evidence there is for the document being in the class.
  - We select the class with the most evidence.

Parameter estimation take 1: Maximum likelihood

- Estimate parameters $\hat{P}(c)$ and $\hat{P}(t_k|c)$ from train data: How?
- Prior:

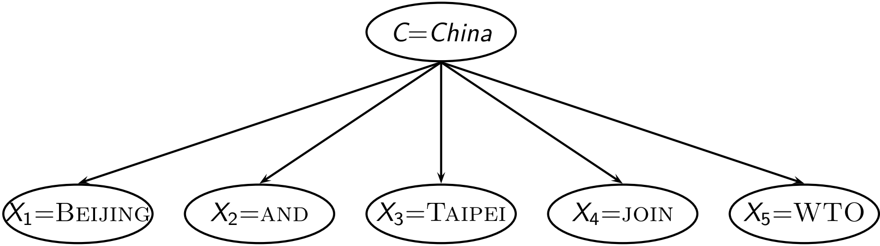
$$\hat{P}(c) = \frac{N_c}{N}$$

- N_c: number of docs in class c; N: total number of docs
- Conditional probabilities:

$$\hat{P}(t|c) = \frac{T_{ct}}{\sum_{t' \in V} T_{ct'}}$$

- T_{ct} is the number of tokens of t in training documents from class c (includes multiple occurrences)
- We've made a Naive Bayes independence assumption here: $\hat{P}(t_k|c) = \hat{P}(t_k|c)$, independent of position

The problem with maximum likelihood estimates: Zeros



$$
\begin{aligned}
P(China|d) \quad \propto \quad & P(China) \cdot P(\text{BEIJING}|China) \cdot P(\text{AND}|China) \\
& \cdot P(\text{TAIPEI}|China) \cdot P(\text{JOIN}|China) \cdot P(\text{WTO}|China)
\end{aligned}
$$

- If WTO never occurs in class China in the train set:

$$
\hat{P}(\text{WTO}|China) = \frac{T_{China,\text{WTO}}}{\sum_{t' \in V} T_{China,t'}} = \frac{0}{\sum_{t' \in V} T_{China,t'}} = 0
$$

The problem with maximum likelihood estimates: Zeros (cont)

- If there are no occurrences of WTO in documents in class China, we get a zero estimate:

$$\hat{P}(\mathrm{WTO}|China) = \frac{T_{China,\mathrm{WTO}}}{\sum_{t' \in V} T_{China,t'}} = 0$$

- $\rightarrow$ We will get $P(China|d) = 0$ for any document that contains WTO!

To avoid zeros: Add-one smoothing

- Before:

$$\hat{P}(t|c) = \frac{T_{ct}}{\sum_{t' \in V} T_{ct'}}$$

- Now: Add one to each count to avoid zeros:

$$\hat{P}(t|c) = \frac{T_{ct} + 1}{\sum_{t' \in V}(T_{ct'} + 1)} = \frac{T_{ct} + 1}{(\sum_{t' \in V} T_{ct'}) + B}$$

- B is the number of bins – in this case the number of different words or the size of the vocabulary $|V| = M$

Naive Bayes: Summary

- Estimate parameters from the training corpus using add-one smoothing
- For a new document, for each class, compute sum of (i) log of prior and (ii) logs of conditional probabilities of the terms
- Assign the document to the class with the largest score

Naive Bayes: Training

TrainMultinomialNB($\mathbb{C}, \mathbb{D}$)

$V \leftarrow$ ExtractVocabulary($\mathbb{D}$)
$N \leftarrow$ CountDocs($\mathbb{D}$)
for each $c \in \mathbb{C}$
do $N_c \leftarrow$ CountDocsInClass($\mathbb{D}, c$)
    $prior[c] \leftarrow N_c/N$
    $text_c \leftarrow$ ConcatenateTextOfAllDocsInClass($\mathbb{D}, c$)
    for each $t \in V$
    do $T_{ct} \leftarrow$ CountTokensOfTerm($text_c, t$)
    for each $t \in V$
    do $condprob[t][c] \leftarrow \frac{T_{ct}+1}{\sum_{t'}(T_{ct'}+1)}$
return $V, prior, condprob$

Naive Bayes: Testing

$\textsc{ApplyMultinomialNB}(\mathbb{C}, V, \textit{prior}, \textit{condprob}, d)$

$W \leftarrow \textsc{ExtractTokensFromDoc}(V, d)$
for each $c \in \mathbb{C}$
do $\textit{score}[c] \leftarrow \log \textit{prior}[c]$
  for each $t \in W$
  do $\textit{score}[c] += \log \textit{condprob}[t][c]$
return $\arg\max_{c \in \mathbb{C}} \textit{score}[c]$

Exercise: Estimate parameters, classify test set

	docID	words in document	in $c = China$?
training set	1	Chinese Beijing Chinese	yes
	2	Chinese Chinese Shanghai	yes
	3	Chinese Macao	yes
	4	Tokyo Japan Chinese	no
test set	5	Chinese Chinese Chinese Tokyo Japan	?

$$\hat{P}(c) = \frac{N_c}{N}$$

$$\hat{P}(t|c) = \frac{T_{ct} + 1}{\sum_{t' \in V}(T_{ct'} + 1)} = \frac{T_{ct} + 1}{(\sum_{t' \in V} T_{ct'}) + B}$$

(B is the number of bins – in this case the number of different words or the size of the vocabulary $|V| = M$)

$$c_{\text{map}} = \arg\max_{c \in \mathbb{C}} [\hat{P}(c) \cdot \prod_{1 \leq k \leq n_d} \hat{P}(t_k|c)]$$

Example: Parameter estimates

Priors: $\hat{P}(c) = 3/4$ and $\hat{P}(\overline{c}) = 1/4$
Conditional probabilities:

$$\begin{aligned}
\hat{P}(\textsc{Chinese}|c) &= (5+1)/(8+6) = 6/14 = 3/7 \\
\hat{P}(\textsc{Tokyo}|c) = \hat{P}(\textsc{Japan}|c) &= (0+1)/(8+6) = 1/14 \\
\hat{P}(\textsc{Chinese}|\overline{c}) &= (1+1)/(3+6) = 2/9 \\
\hat{P}(\textsc{Tokyo}|\overline{c}) = \hat{P}(\textsc{Japan}|\overline{c}) &= (1+1)/(3+6) = 2/9
\end{aligned}$$

The denominators are $(8+6)$ and $(3+6)$ because the lengths of $text_c$ and $text_{\overline{c}}$ are 8 and 3, respectively, and because the constant B is 6 as the vocabulary consists of six terms.

Example: Classification

$$\hat{P}(c|d_5) \quad \propto \quad 3/4 \cdot (3/7)^3 \cdot 1/14 \cdot 1/14 \approx 0.0003$$
$$\hat{P}(\overline{c}|d_5) \quad \propto \quad 1/4 \cdot (2/9)^3 \cdot 2/9 \cdot 2/9 \approx 0.0001$$

Thus, the classifier assigns the test document to $c = China$.
The reason for this classification decision is that the three occurrences of the positive indicator CHINESE in d_5 outweigh the occurrences of the two negative indicators JAPAN and TOKYO.

Time complexity of Naive Bayes

mode	time complexity
training	$\Theta(\lvert\mathbb{D}\rvert L_{\mathrm{ave}} + \lvert\mathbb{C}\rvert\lvert V\rvert)$
testing	$\Theta(L_{\mathrm{a}} + \lvert\mathbb{C}\rvert M_{\mathrm{a}}) = \Theta(\lvert\mathbb{C}\rvert M_{\mathrm{a}})$

- L_{ave}: average length of a training doc, L_{a}: length of the test doc, M_{a}: number of distinct terms in the test doc, $\mathbb{D}$: training set, V: vocabulary, $\mathbb{C}$: set of classes
- $\Theta(\lvert\mathbb{D}\rvert L_{\mathrm{ave}})$ is the time it takes to compute all counts.
- $\Theta(\lvert\mathbb{C}\rvert\lvert V\rvert)$ is the time it takes to compute the parameters from the counts.
- Generally: $\lvert\mathbb{C}\rvert\lvert V\rvert < \lvert\mathbb{D}\rvert L_{\mathrm{ave}}$
- Test time is also linear (in the length of the test document).
- Thus: Naive Bayes is linear in the size of the training set (training) and the test document (testing). This is optimal.

Naive Bayes: Analysis

- Now we want to gain a better understanding of the properties of Naive Bayes.
- We will formally derive the classification rule ...
- ...and make our assumptions explicit.

Derivation of Naive Bayes rule

We want to find the class that is most likely given the document:

$$c_{\text{map}} = \arg\max_{c \in \mathbb{C}} P(c|d)$$

Apply Bayes rule $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$:

$$c_{\text{map}} = \arg\max_{c \in \mathbb{C}} \frac{P(d|c)P(c)}{P(d)}$$

Drop denominator since $P(d)$ is the same for all classes:

$$c_{\text{map}} = \arg\max_{c \in \mathbb{C}} P(d|c)P(c)$$

Too many parameters / sparseness

$$\begin{aligned} c_{\text{map}} &= \arg\max_{c \in \mathbb{C}} \; P(d|c)P(c) \\ &= \arg\max_{c \in \mathbb{C}} P(\langle t_1, \ldots, t_k, \ldots, t_{n_d} \rangle | c) P(c) \end{aligned}$$

- There are too many parameters $P(\langle t_1, \ldots, t_k, \ldots, t_{n_d} \rangle | c)$, one for each unique combination of a class and a sequence of words.
- We would need a very, very large number of training examples to estimate that many parameters.
- This is the problem of data sparseness.

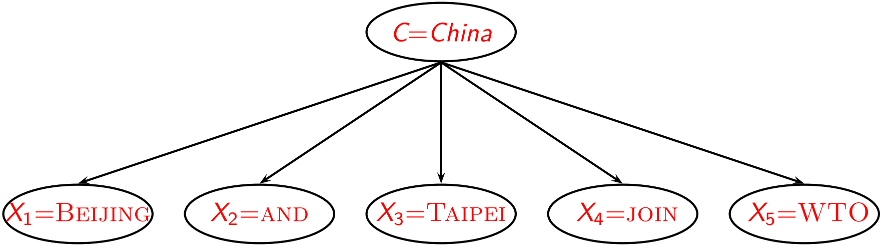
Naive Bayes conditional independence assumption

To reduce the number of parameters to a manageable size, we make the Naive Bayes conditional independence assumption:

$$P(d|c) = P(\langle t_1, \ldots, t_{n_d} \rangle | c) = \prod_{1 \leq k \leq n_d} P(X_k = t_k | c)$$

We assume that the probability of observing the conjunction of attributes is equal to the product of the individual probabilities $P(X_k = t_k|c)$.
Recall from earlier the estimates for these conditional probabilities: $\hat{P}(t|c) = \frac{T_{ct}+1}{(\sum_{t' \in V} T_{ct'})+B}$

Generative model



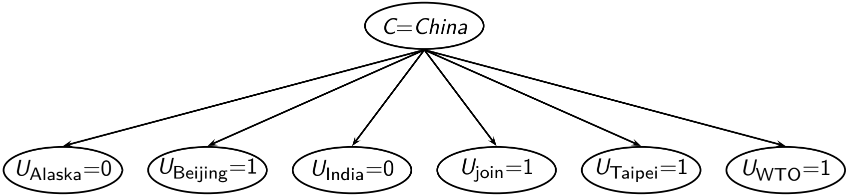
$P(c|d) \propto P(c) \prod_{1 \leq k \leq n_d} P(t_k|c)$

- Generate a class with probability $P(c)$
- Generate each of the words (in their respective positions), conditional on the class, but independent of each other, with probability $P(t_k|c)$
- To classify docs, we "reengineer" this process and find the class that is most likely to have generated the doc.

Second independence assumption

- $\hat{P}(X_{k_1} = t|c) = \hat{P}(X_{k_2} = t|c)$
- For example, for a document in the class *UK*, the probability of generating QUEEN in the first position of the document is the same as generating it in the last position.
- The two independence assumptions amount to the bag of words model.

A different Naive Bayes model: Bernoulli model



Violation of Naive Bayes independence assumptions

- Conditional independence:

$$P(\langle t_1, \ldots, t_{n_d} \rangle | c) = \prod_{1 \le k \le n_d} P(X_k = t_k | c)$$

- Positional independence:
- $\hat{P}(X_{k_1} = t|c) = \hat{P}(X_{k_2} = t|c)$
- The independence assumptions do not really hold of documents written in natural language.
- Exercise
  - Examples for why conditional independence assumption is not really true?
  - Examples for why positional independence assumption is not really true?
- How can Naive Bayes work if it makes such inappropriate assumptions?

Why does Naive Bayes work?

- Naive Bayes can work well even though conditional independence assumptions are badly violated.
- Example:

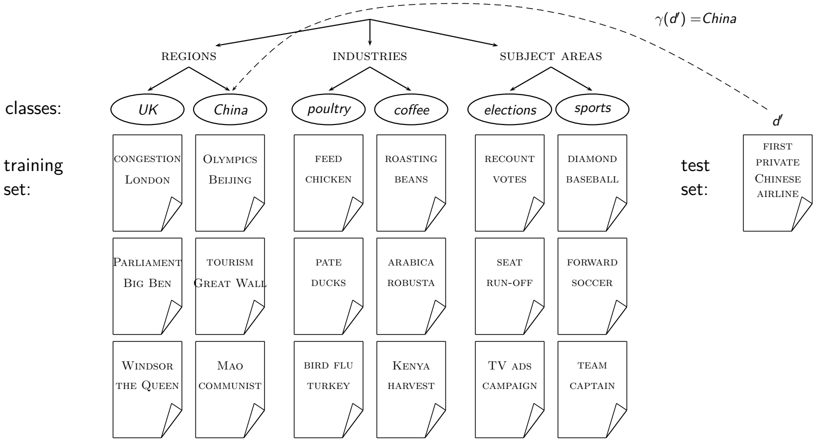
	c_1	c_2	class selected
true probability $P(c\|d)$	0.6	0.4	c_1
$\hat{P}(c)\prod_{1\leq k\leq n_d}\hat{P}(t_k\|c)$	0.00099	0.00001	
NB estimate $\hat{P}(c\|d)$	0.99	0.01	c_1

- Double counting of evidence causes underestimation (0.01) and overestimation (0.99).
- Classification is about predicting the correct class and not about accurately estimating probabilities.
- Naive Bayes is terrible for correct estimation ...
- ...but if often performs well at accurate prediction (choosing the correct class).

Naive Bayes is not so naive

- Naive Bayes has won some bakeoffs (e.g., KDD-CUP 97)
- More robust to nonrelevant features than some more complex learning methods
- More robust to concept drift (changing of definition of class over time) than some more complex learning methods
- Better than methods like decision trees when we have many equally important features
- A good dependable baseline for text classification (but not the best)
- Optimal if independence assumptions hold (never true for text, but true for some domains)
- Very fast
- Low storage requirements

Evaluation on Reuters



Example: The Reuters collection

symbol	statistic	value
N	documents	800,000
L	avg. # word tokens per document	200
M	word types	400,000

type of class	number	examples
region	366	UK, China
industry	870	poultry, coffee
subject area	126	elections, sports

A Reuters document

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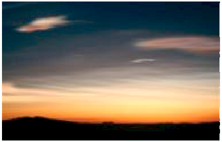


SYDNEY (Reuters) - Rare, mother-of-pearl colored clouds caused by extreme weather conditions above Antarctica are a possible indication of global warming, Australian scientists said on Tuesday.

Known as nacreous clouds, the spectacular formations showing delicate wisps of colors were photographed in the sky over an Australian

Evaluating classification

- Evaluation must be done on test data that are independent of the training data, i.e., training and test sets are disjoint.
- It's easy to get good performance on a test set that was available to the learner during training (e.g., just memorize the test set).
- Measures: Precision, recall, F_1, classification accuracy

Precision P and recall R

	in the class	not in the class
predicted to be in the class	true positives (TP)	false positives (FP)
predicted to not be in the class	false negatives (FN)	true negatives (TN)

TP, FP, FN, TN are counts of documents. The sum of these four counts is the total number of documents.

$$\begin{aligned} \text{precision:}P &= TP/(TP+FP) \\ \text{recall:}R &= TP/(TP+FN) \end{aligned}$$

A combined measure: F

- F_1 allows us to trade off precision against recall.
- $$F_1 = \frac{1}{\frac{1}{2}\frac{1}{P} + \frac{1}{2}\frac{1}{R}} = \frac{2PR}{P+R}$$
- This is the harmonic mean of P and R: $\frac{1}{F} = \frac{1}{2}(\frac{1}{P} + \frac{1}{R})$

Averaging: Micro vs. Macro

- We now have an evaluation measure (F_1) for one class.
- But we also want a single number that measures the aggregate performance over all classes in the collection.
- Macroaveraging
  - Compute F_1 for each of the C classes
  - Average these C numbers
- Microaveraging
  - Compute TP, FP, FN for each of the C classes
  - Sum these C numbers (e.g., all TP to get aggregate TP)
  - Compute F_1 for aggregate TP, FP, FN

F_1 scores for Naive Bayes vs. other methods

(a)	NB	Rocchio	kNN		SVM
micro-avg-L (90 classes)	80	85	86		89
macro-avg (90 classes)	47	59	60		60

(b)	NB	Rocchio	kNN	trees	SVM
earn	96	93	97	98	98
acq	88	65	92	90	94
money-fx	57	47	78	66	75
grain	79	68	82	85	95
crude	80	70	86	85	89
trade	64	65	77	73	76
interest	65	63	74	67	78
ship	85	49	79	74	86
wheat	70	69	77	93	92
corn	65	48	78	92	90
micro-avg (top 10)	82	65	82	88	92
micro-avg-D (118 classes)	75	62	n/a	n/a	87

Naive Bayes does pretty well, but some methods beat it consistently (e.g., SVM).

Feature selection

- In text classification, we usually represent documents in a high-dimensional space, with each dimension corresponding to a term.
- In this lecture: axis = dimension = word = term = feature
- Many dimensions correspond to rare words.
- Rare words can mislead the classifier.
- Rare misleading features are called noise features.
- Eliminating noise features from the representation increases efficiency and effectiveness of text classification.
- Eliminating features is called feature selection.

Example for a noise feature

- Let's say we're doing text classification for the class *China*.
- Suppose a rare term, say ARACHNOCENTRIC, has no information about *China* ...
- ...but all instances of ARACHNOCENTRIC happen to occur in *China* documents in our training set.
- Then we may learn a classifier that incorrectly interprets ARACHNOCENTRIC as evidence for the class *China*.
- Such an incorrect generalization from an accidental property of the training set is called overfitting.
- Feature selection reduces overfitting and improves the accuracy of the classifier.

Basic feature selection algorithm

$\textsc{SelectFeatures}(\mathbb{D}, c, k)$
$V \leftarrow \textsc{ExtractVocabulary}(\mathbb{D})$
$L \leftarrow []$
for each $t \in V$
do $A(t, c) \leftarrow \textsc{ComputeFeatureUtility}(\mathbb{D}, t, c)$
    $\textsc{Append}(L, \langle A(t, c), t \rangle)$
return $\textsc{FeaturesWithLargestValues}(L, k)$

How do we compute A, the feature utility?

Different feature selection methods

- A feature selection method is mainly defined by the feature utility measure it employs
- Feature utility measures:
  - Frequency – select the most frequent terms
  - Mutual information – select the terms with the highest mutual information
  - Mutual information is also called information gain in this context.
  - Chi-square (see book)

Mutual information

- Compute the feature utility $A(t, c)$ as the mutual information (MI) of term t and class c.
- MI tells us "how much information" the term contains about the class and vice versa.
- For example, if a term's occurrence is independent of the class (same proportion of docs within/without class contain the term), then MI is 0.
- Definition:

$$I(U; C) = \sum_{e_t \in \{1,0\}} \sum_{e_c \in \{1,0\}} P(U = e_t, C = e_c) \log_2 \frac{P(U = e_t, C = e_c)}{P(U = e_t) P(C = e_c)}$$

How to compute MI values

- Based on maximum likelihood estimates, the formula we actually use is:

$$
\begin{aligned}
I(U;C) \quad = \quad & \frac{N_{11}}{N}\log_2\frac{NN_{11}}{N_{1.}N_{.1}} + \frac{N_{01}}{N}\log_2\frac{NN_{01}}{N_{0.}N_{.1}} \\
& + \frac{N_{10}}{N}\log_2\frac{NN_{10}}{N_{1.}N_{.0}} + \frac{N_{00}}{N}\log_2\frac{NN_{00}}{N_{0.}N_{.0}}
\end{aligned}
$$

- N_{10}: number of documents that contain t ($e_t = 1$) and are not in c ($e_c = 0$); N_{11}: number of documents that contain t ($e_t = 1$) and are in c ($e_c = 1$); N_{01}: number of documents that do not contain t ($e_t = 1$) and are in c ($e_c = 1$); N_{00}: number of documents that do not contain t ($e_t = 1$) and are not in c ($e_c = 1$); $N = N_{00} + N_{01} + N_{10} + N_{11}$.

How to compute MI values (2)

- Alternative way of computing MI:

$$I(U;C) = \sum_{e_t \in \{1,0\}} \sum_{e_c \in \{1,0\}} P(U = e_t, C = e_c) \log_2 \frac{N(U = e_t, C = e_c)}{E(U = e_t) E(C = e_c)}$$

- $N(U = e_t, C = e_c)$ is the count of documents with values e_t and e_c .
- $E(U = e_t, C = e_c)$ is the expected count of documents with values e_t and e_c if we assume that the two random variables are independent.

MI example for *poultry*/EXPORT in Reuters

	$e_c = e_{poultry} = 1$	$e_c = e_{poultry} = 0$
$e_t = e_{\text{EXPORT}} = 1$	$N_{11} = 49$	$N_{10} = 27{,}652$
$e_t = e_{\text{EXPORT}} = 0$	$N_{01} = 141$	$N_{00} = 774{,}106$

Plug these values into formula:

$$\begin{aligned} I(U;C) &= \frac{49}{801{,}948}\log_2\frac{801{,}948\cdot 49}{(49+27{,}652)(49+141)} \\ &+\frac{141}{801{,}948}\log_2\frac{801{,}948\cdot 141}{(141+774{,}106)(49+141)} \\ &+\frac{27{,}652}{801{,}948}\log_2\frac{801{,}948\cdot 27{,}652}{(49+27{,}652)(27{,}652+774{,}106)} \\ &+\frac{774{,}106}{801{,}948}\log_2\frac{801{,}948\cdot 774{,}106}{(141+774{,}106)(27{,}652+774{,}106)} \\ &\approx 0.000105 \end{aligned}$$

MI feature selection on Reuters

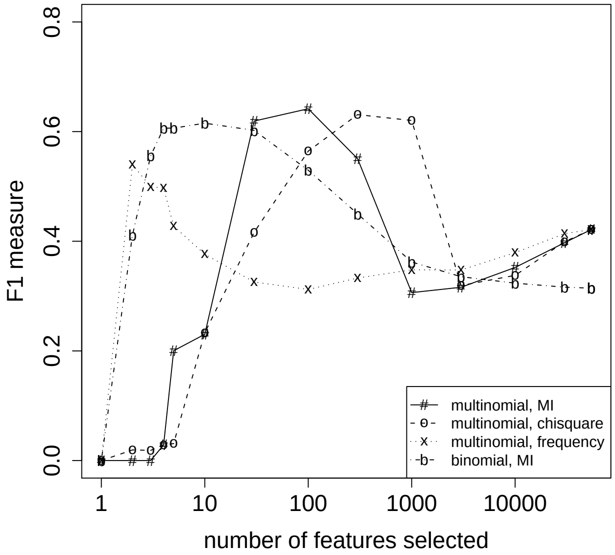
Class: *coffee*

term	MI
COFFEE	0.0111
BAGS	0.0042
GROWERS	0.0025
KG	0.0019
COLOMBIA	0.0018
BRAZIL	0.0016
EXPORT	0.0014
EXPORTERS	0.0013
EXPORTS	0.0013
CROP	0.0012

Class: *sports*

term	MI
SOCCER	0.0681
CUP	0.0515
MATCH	0.0441
MATCHES	0.0408
PLAYED	0.0388
LEAGUE	0.0386
BEAT	0.0301
GAME	0.0299
GAMES	0.0284
TEAM	0.0264

Naive Bayes: Effect of feature selection



(multinomial = multinomial Naive Bayes, binomial = Bernoulli Naive Bayes)

Feature selection for Naive Bayes

- In general, feature selection is necessary for Naive Bayes to get decent performance.
- Also true for many other learning methods in text classification: you need feature selection for optimal performance.

Exercise

(i) Compute the "export"/POULTRY contingency table for the "Kyoto"/JAPAN in the collection given below. (ii) Make up a contingency table for which MI is 0 – that is, term and class are independent of each other.

"export"/POULTRY table:

	$e_c = e_{poultry} = 1$	$e_c = e_{poultry} = 0$
$e_t = e_{\text{EXPORT}} = 1$	$N_{11} = 49$	$N_{10} = 27{,}652$
$e_t = e_{\text{EXPORT}} = 0$	$N_{01} = 141$	$N_{00} = 774{,}106$

Collection:

	docID	words in document	in c = Japan?
training set	1	Kyoto Osaka Taiwan	yes
	2	Japan Kyoto	yes
	3	Taipei Taiwan	no
	4	Macao Taiwan Shanghai	no
	5	London	no

Feature selection: MI for *poultry*/EXPORT

Goal of feature selection: eleminate noise and useless features for better effectiveness and efficiency

	$e_c = e_{poultry} = 1$	$e_c = e_{poultry} = 0$
$e_t = e_{\mathrm{EXPORT}} = 1$	$N_{11} = 49$	$N_{10} = 27{,}652$
$e_t = e_{\mathrm{EXPORT}} = 0$	$N_{01} = 141$	$N_{00} = 774{,}106$

Plug these values into formula:

$$\begin{aligned}
I(U;C) &= \frac{49}{801{,}948}\log_2\frac{801{,}948\cdot 49}{(49+27{,}652)(49+141)}\\
&\quad+\frac{141}{801{,}948}\log_2\frac{801{,}948\cdot 141}{(141+774{,}106)(49+141)}\\
&\quad+\frac{27{,}652}{801{,}948}\log_2\frac{801{,}948\cdot 27{,}652}{(49+27{,}652)(27{,}652+774{,}106)}\\
&\quad+\frac{774{,}106}{801{,}948}\log_2\frac{801{,}948\cdot 774{,}106}{(141+774{,}106)(27{,}652+774{,}106)}\\
&\approx 0.000105
\end{aligned}$$

Feature selection for Reuters classes coffee and sports

Class: *coffee*

term	MI
COFFEE	0.0111
BAGS	0.0042
GROWERS	0.0025
KG	0.0019
COLOMBIA	0.0018
BRAZIL	0.0016
EXPORT	0.0014
EXPORTERS	0.0013
EXPORTS	0.0013
CROP	0.0012

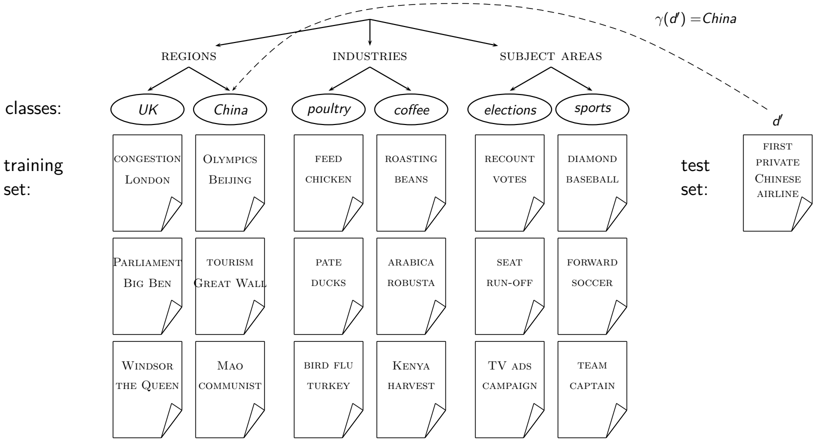
Class: *sports*

term	MI
SOCCER	0.0681
CUP	0.0515
MATCH	0.0441
MATCHES	0.0408
PLAYED	0.0388
LEAGUE	0.0386
BEAT	0.0301
GAME	0.0299
GAMES	0.0284
TEAM	0.0264

Recall vector space representation

- Each document is a vector, one component for each term.
- Terms are axes.
- High dimensionality: 100,000s of dimensions
- Normalize vectors (documents) to unit length
- How can we do classification in this space?

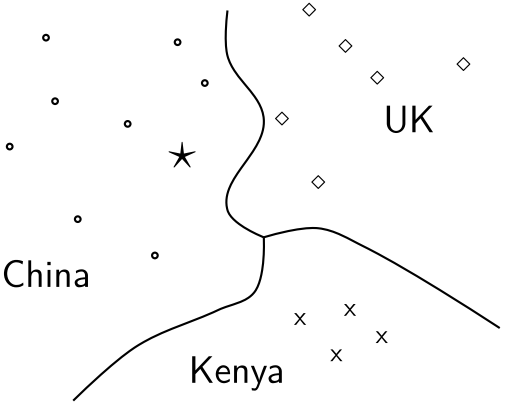
Basic text classification setup



Vector space classification

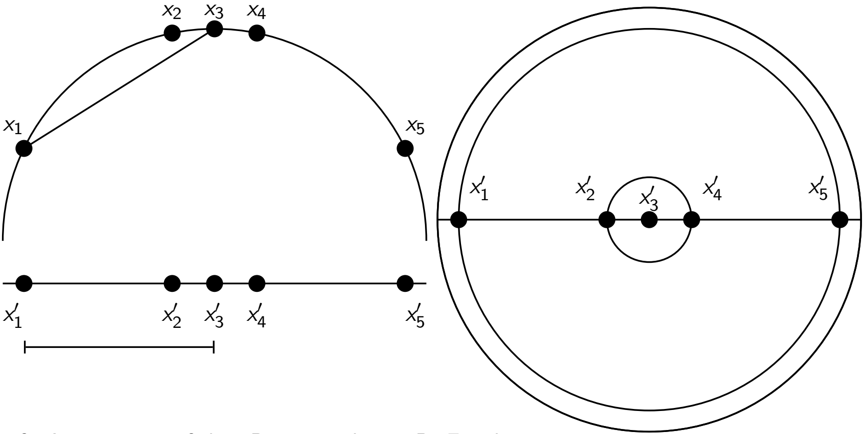
- As before, the training set is a set of documents, each labeled with its class.
- In vector space classification, this set corresponds to a labeled set of points or vectors in the vector space.
- Premise 1: Documents in the same class form a contiguous region.
- Premise 2: Documents from different classes don't overlap.
- We define lines, surfaces, hypersurfaces to divide regions.

Classes in the vector space



Should the document $\star$ be assigned to *China*, *UK* or *Kenya*?
Find separators between the classes
Based on these separators: $\star$ should be assigned to *China*
How do we find separators that do a good job at classifying new documents like $\star$? – Main topic of today

Aside: 2D/3D graphs can be misleading



Left: A projection of the 2D semicircle to 1D. For the points x_1, x_2, x_3, x_4, x_5 at x coordinates $-0.9, -0.2, 0, 0.2, 0.9$ the distance $|x_2 x_3| \approx 0.201$ only differs by 0.5% from $|x'_2 x'_3| = 0.2$; but $|x_1 x_3|/|x'_1 x'_3| = d_{\text{true}}/d_{\text{projected}} \approx 1.06/0.9 \approx 1.18$ is an example of a large distortion (18%) when projecting a large area. *Right:* The corresponding projection of the 3D hemisphere to 2D.

Relevance feedback

- In relevance feedback, the user marks documents as relevant/nonrelevant.
- Relevant/nonrelevant can be viewed as classes or categories.
- For each document, the user decides which of these two classes is correct.
- The IR system then uses these class assignments to build a better query ("model") of the information need ...
- ...and returns better documents.
- Relevance feedback is a form of text classification.

Using Rocchio for vector space classification

- The principal difference between relevance feedback and text classification:
  - The training set is given as part of the input in text classification.
  - It is interactively created in relevance feedback.

Rocchio classification: Basic idea

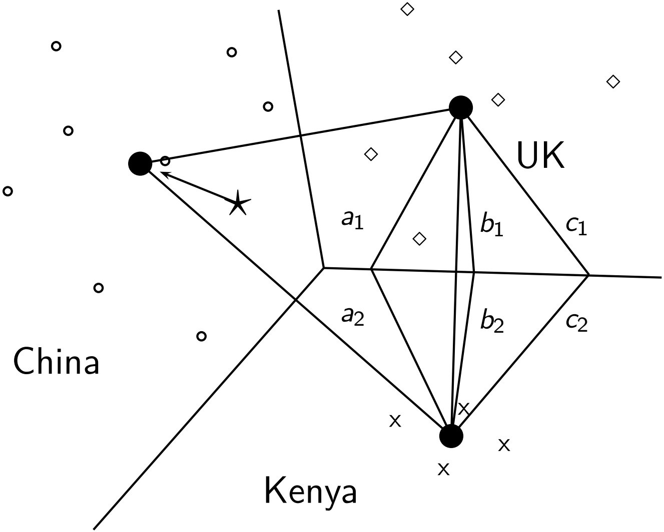
- Compute a centroid for each class
  - The centroid is the average of all documents in the class.
- Assign each test document to the class of its closest centroid.

Recall definition of centroid

$$\vec{\mu}(c) = \frac{1}{|D_c|} \sum_{d \in D_c} \vec{v}(d)$$

where D_c is the set of all documents that belong to class c and $\vec{v}(d)$ is the vector space representation of d.

Rocchio illustrated: $a_1 = a_2, b_1 = b_2, c_1 = c_2$



Rocchio algorithm

$\textsc{TrainRocchio}(\mathbb{C}, \mathbb{D})$

for each $c_j \in \mathbb{C}$
do $D_j \leftarrow \{d : \langle d, c_j \rangle \in \mathbb{D}\}$
$\quad\vec{\mu}_j \leftarrow \frac{1}{|D_j|} \sum_{d \in D_j} \vec{v}(d)$
return $\{\vec{\mu}_1, \ldots, \vec{\mu}_J\}$

$\textsc{ApplyRocchio}(\{\vec{\mu}_1, \ldots, \vec{\mu}_J\}, d)$

return $\arg\min_j |\vec{\mu}_j - \vec{v}(d)|$

Rocchio properties

- Rocchio forms a simple representation for each class: the centroid
  - We can interpret the centroid as the prototype of the class.
- Classification is based on similarity to / distance from centroid/prototype.
- Does not guarantee that classifications are consistent with the training data!

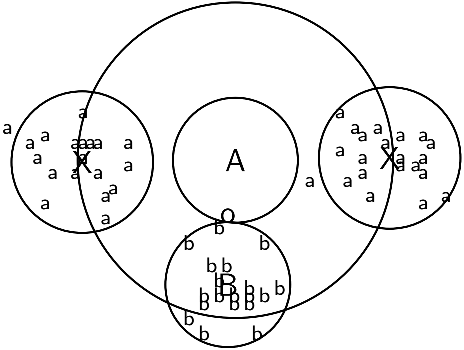
Time complexity of Rocchio

mode	time complexity
training	$\Theta(\lvert\mathbb{D}\rvert L_{\text{ave}} + \lvert\mathbb{C}\rvert\lvert V\rvert) \approx \Theta(\lvert\mathbb{D}\rvert L_{\text{ave}})$
testing	$\Theta(L_{\text{a}} + \lvert\mathbb{C}\rvert M_{\text{a}}) \approx \Theta(\lvert\mathbb{C}\rvert M_{\text{a}})$

Rocchio vs. Naive Bayes

- In many cases, Rocchio performs worse than Naive Bayes.
- One reason: Rocchio does not handle nonconvex, multimodal classes correctly.

Rocchio cannot handle nonconvex, multimodal classes



Exercise: Why is Rocchio not expected to do well for the classification task a vs. b here?

- A is centroid of the a's, B is centroid of the b's.
- The point o is closer to A than to B.
- But o is a better fit for the b class.
- A is a multimodal class with two prototypes.
- But in Rocchio we only have one prototype

kNN classification

- kNN classification is another vector space classification method.
- It also is very simple and easy to implement.
- kNN is more accurate (in most cases) than Naive Bayes and Rocchio.
- If you need to get a pretty accurate classifier up and running in a short time ...
- ...and you don't care about efficiency that much ...
- ...use kNN.

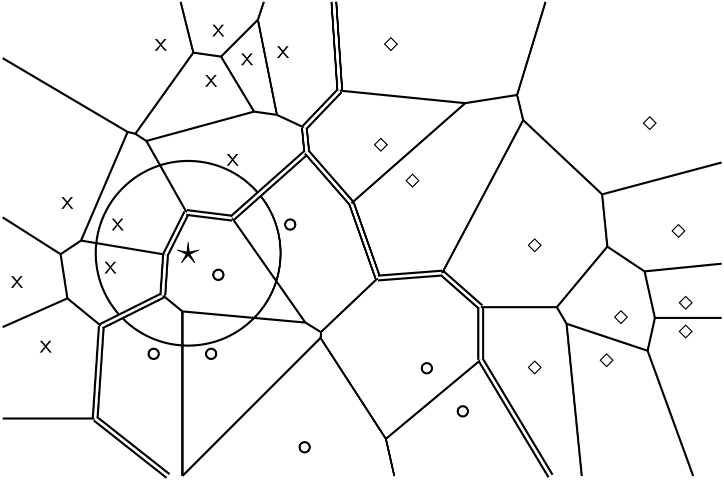
kNN classification

- kNN = k nearest neighbors
- kNN classification rule for $k = 1$ (1NN): Assign each test document to the class of its nearest neighbor in the training set.
- 1NN is not very robust – one document can be mislabeled or atypical.
- kNN classification rule for $k > 1$ (kNN): Assign each test document to the majority class of its k nearest neighbors in the training set.
- Rationale of kNN: contiguity hypothesis
  - We expect a test document d to have the same label as the training documents located in the local region surrounding d.

Probabilistic kNN

- Probabilistic version of kNN: $P(c|d) =$ fraction of k neighbors of d that are in c
- kNN classification rule for probabilistic kNN: Assign d to class c with highest $P(c|d)$

kNN is based on Voronoi tessellation



kNN algorithm

$\textsc{Train-kNN}(\mathbb{C}, \mathbb{D})$

$\mathbb{D}' \leftarrow \textsc{Preprocess}(\mathbb{D})$

$k \leftarrow \textsc{Select-k}(\mathbb{C}, \mathbb{D}')$

return $\mathbb{D}', k$

$\textsc{Apply-kNN}(\mathbb{D}', k, d)$

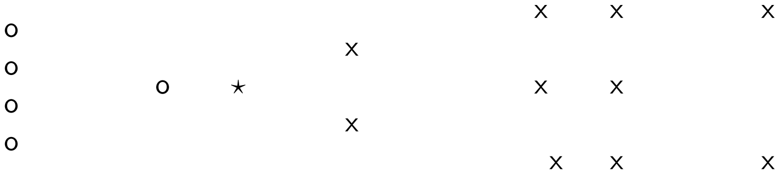
$S_k \leftarrow \textsc{ComputeNearestNeighbors}(\mathbb{D}', k, d)$

for each $c_j \in \mathbb{C}(\mathbb{D}')$

do $p_j \leftarrow |S_k \cap c_j| / k$

return $\arg\max_j p_j$

Exercise



How is star classified by:
(i) 1-NN (ii) 3-NN (iii) 9-NN (iv) 15-NN (v) Rocchio?

Time complexity of kNN

kNN with preprocessing of training set

training $\Theta(|\mathbb{D}|L_{\mathrm{ave}})$

testing $\Theta(L_{\mathrm{a}} + |\mathbb{D}|M_{\mathrm{ave}}M_{\mathrm{a}}) = \Theta(|\mathbb{D}|M_{\mathrm{ave}}M_{\mathrm{a}})$

- kNN test time proportional to the size of the training set!
- The larger the training set, the longer it takes to classify a test document.
- kNN is inefficient for very large training sets.
- Question: Can we divide up the training set into regions, so that we only have to search in one region to do kNN classification for a given test document? (which perhaps would give us better than linear time complexity)

Curse of dimensionality

- Our intuitions about space are based on the 3D world we live in.
- Intuition 1: some things are close by, some things are distant.
- Intuition 2: we can carve up space into areas such that: within an area things are close, distances between areas are large.
- These two intuitions don't necessarily hold for high dimensions.
- In particular: for a set of k uniformly distributed points, let dmin be the smallest distance between any two points and dmax be the largest distance between any two points.
- Then

$$\lim_{d \to \infty} \frac{\text{dmax} - \text{dmin}}{\text{dmin}} = 0$$

Curse of dimensionality: Simulation

- Simulate

$$\lim_{d \to \infty} \frac{\text{dmax} - \text{dmin}}{\text{dmin}} = 0$$

- Pick a dimensionality d
- Generate 10 random points in the d-dimensional hypercube (uniform distribution)
- Compute all 45 distances
- Compute $\frac{\text{dmax}-\text{dmin}}{\text{dmin}}$
- We see that intuition 1 (some things are close, others are distant) is not true for high dimensions.

Intuition 2: Space can be carved up

- Intuition 2: we can carve up space into areas such that: within an area things are close, distances between areas are large.
- If this is true, then we have a simple and efficient algorithm for kNN.
- To find the k closest neighbors of data point $< x_1, x_2, \ldots, x_d >$ do the following.
- Using binary search find all data points whose first dimension is in $[x_1 - \epsilon, x_1 + \epsilon]$. This is $O(\log n)$ where n is the number of data points.
- Do this for each dimension, then intersect the d subsets.

Intuition 2: Space can be carved up

- Size of data set $n = 100$
- Again, assume uniform distribution in hypercube
- Set $\epsilon = 0.05$: we will look in an interval of length 0.1 for neighbors on each dimension.
- What is the probability that the nearest neighbor of a new data point $\vec{x}$ is in this neighborhood in $d = 1$ dimension?
- for $d = 1$: $1 - (1 - 0.1)^{100} \approx 0.99997$
- In $d = 2$ dimensions?
- for $d = 2$: $1 - (1 - 0.1^2)^{100} \approx 0.63$
- In $d = 3$ dimensions?
- for $d = 3$: $1 - (1 - 0.1^3)^{100} \approx 0.095$
- In $d = 4$ dimensions?
- for $d = 4$: $1 - (1 - 0.1^4)^{100} \approx 0.0095$
- In $d = 5$ dimensions?
- for $d = 5$: $1 - (1 - 0.1^5)^{100} \approx 0.0009995$

Intuition 2: Space can be carved up

- In $d = 5$ dimensions?
- for $d = 5$: $1 - (1 - 0.1^5)^{100} \approx 0.0009995$
- In other words: with enough dimensions, there is only one "local" region that will contain the nearest neighbor with high certainty: the entire search space.
- We cannot carve up high-dimensional space into neat neighborhoods ...
- ...unless the "true" dimensionality is much lower than d.

kNN: Discussion

- No training necessary
  - But linear preprocessing of documents is as expensive as training Naive Bayes.
  - We always preprocess the training set, so in reality training time of kNN is linear.
- kNN is very accurate if training set is large.
- Optimality result: asymptotically zero error if Bayes rate is zero.
- But kNN can be very inaccurate if training set is small.

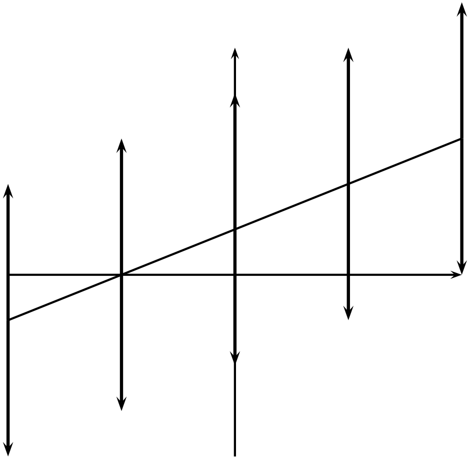
Linear classifiers

- Definition:
  - A linear classifier computes a linear combination or weighted sum $\sum_i w_i x_i$ of the feature values.
  - Classification decision: $\sum_i w_i x_i > \theta$?
  - ...where θ (the threshold) is a parameter.
- (First, we only consider binary classifiers.)
- Geometrically, this corresponds to a line (2D), a plane (3D) or a hyperplane (higher dimensionalities), the separator.
- We find this separator based on training set.
- Methods for finding separator: Perceptron, Rocchio, Naive Bayes – as we will explain on the next slides
- Assumption: The classes are linearly separable.

A linear classifier in 1D

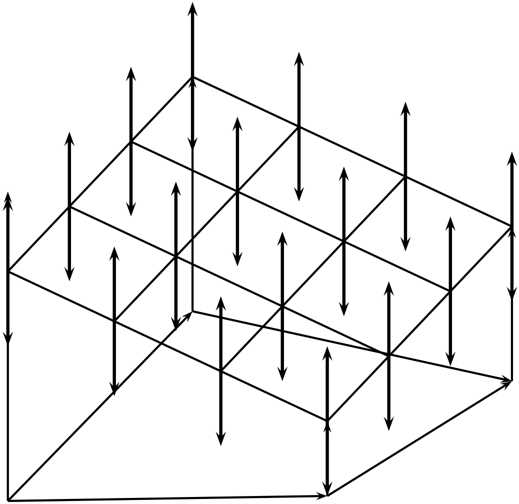
- A linear classifier in 1D is a point described by the equation $w_1 d_1 = \theta$
- The point at θ / w_1
- Points (d_1) with $w_1 d_1 \geq \theta$ are in the class c.
- Points (d_1) with $w_1 d_1 < \theta$ are in the complement class $\bar{c}$.

A linear classifier in 2D



- A linear classifier in 2D is a line described by the equation $w_1 d_1 + w_2 d_2 = \theta$
- Example for a 2D linear classifier
- Points $(d_1 \ d_2)$ with $w_1 d_1 + w_2 d_2 \geq \theta$ are in the class c.
- Points $(d_1 \ d_2)$ with $w_1 d_1 + w_2 d_2 < \theta$ are in the complement class $\overline{c}$.

A linear classifier in 3D



- A linear classifier in 3D is a plane described by the equation $w_1 d_1 + w_2 d_2 + w_3 d_3 = \theta$
- Example for a 3D linear classifier
- Points $(d_1\ d_2\ d_3)$ with $w_1 d_1 + w_2 d_2 + w_3 d_3 \geq \theta$ are in the class c.
- Points $(d_1\ d_2\ d_3)$ with $w_1 d_1 + w_2 d_2 + w_3 d_3 < \theta$ are in the complement class $\bar{c}$.

Rocchio as a linear classifier

- Rocchio is a linear classifier defined by:

$$\sum_{i=1}^{M} w_i d_i = \vec{w}\vec{d} = \theta$$

where $\vec{w}$ is the normal vector $\vec{\mu}(c_1) - \vec{\mu}(c_2)$ and $\theta = 0.5 * (|\vec{\mu}(c_1)|^2 - |\vec{\mu}(c_2)|^2)$.

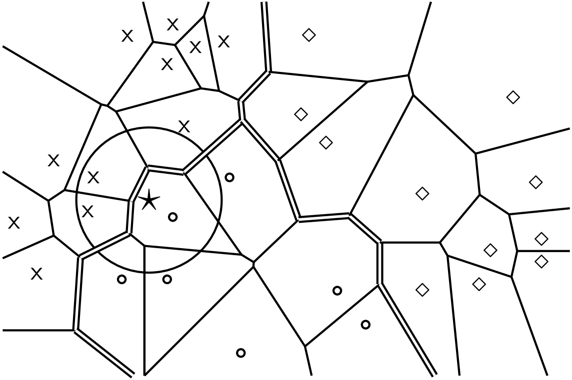
Naive Bayes as a linear classifier

Multinomial Naive Bayes is a linear classifier (in log space) defined by:

$$\sum_{i=1}^{M} w_i d_i = \theta$$

where $w_i = \log[\hat{P}(t_i|c)/\hat{P}(t_i|\bar{c})]$, $d_i =$ number of occurrences of t_i in d, and $\theta = -\log[\hat{P}(c)/\hat{P}(\bar{c})]$. Here, the index i, $1 \leq i \leq M$, refers to terms of the vocabulary (not to positions in d as k did in our original definition of Naive Bayes)

kNN is not a linear classifier



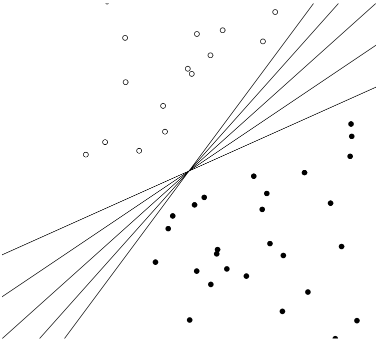
- Classification decision based on majority of k nearest neighbors.
- The decision boundaries between classes are piecewise linear ...
- ...but they are in general not linear classifiers that can be described as $\sum_{i=1}^{M} w_i d_i = \theta$.

Example of a linear two-class classifier

t_i	w_i	d_{1i}	d_{2i}	t_i	w_i	d_{1i}	d_{2i}
prime	0.70	0	1	dlrs	-0.71	1	1
rate	0.67	1	0	world	-0.35	1	0
interest	0.63	0	0	sees	-0.33	0	0
rates	0.60	0	0	year	-0.25	0	0
discount	0.46	1	0	group	-0.24	0	0
bundesbank	0.43	0	0	dlr	-0.24	0	0

- This is for the class *interest* in Reuters-21578.
- For simplicity: assume a simple 0/1 vector representation
- d_1: "rate discount dlrs world"
- d_2: "prime dlrs"
- $\theta = 0$
- Exercise: Which class is d_1 assigned to? Which class is d_2 assigned to?
- We assign document $\vec{d}_1$ "rate discount dlrs world" to *interest* since $\vec{w}^T\vec{d}_1 = 0.67 \cdot 1 + 0.46 \cdot 1 + (-0.71) \cdot 1 + (-0.35) \cdot 1 = 0.07 > 0 = \theta$.
- We assign $\vec{d}_2$ "prime dlrs" to the complement class (not in *interest*) since $\vec{w}^T\vec{d}_2 = -0.01 \leq \theta$.

Which hyperplane?



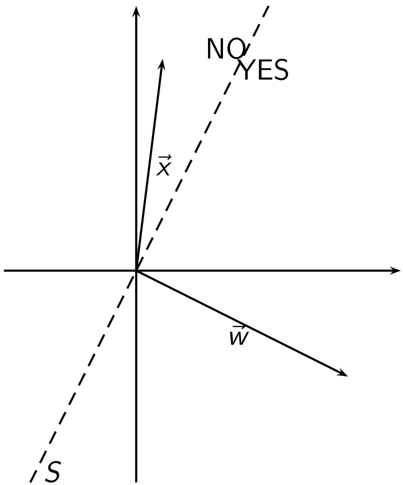
Learning algorithms for vector space classification

- In terms of actual computation, there are two types of learning algorithms.
- (i) Simple learning algorithms that estimate the parameters of the classifier directly from the training data, often in one linear pass.
  - Naive Bayes, Rocchio, kNN are all examples of this.
- (ii) Iterative algorithms
  - Support vector machines
  - Perceptron (example available as PDF on website: http://cislmu.org)
- The best performing learning algorithms usually require iterative learning.

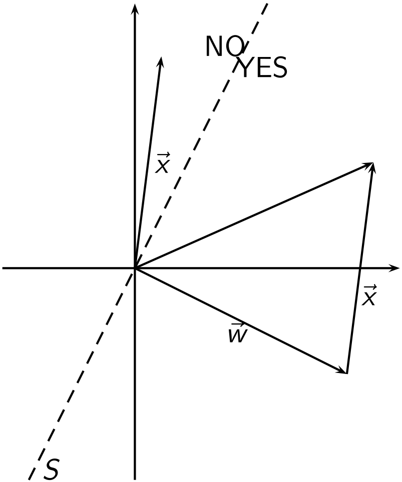
Perceptron update rule

- Randomly initialize linear separator $\vec{w}$
- Do until convergence:
  - Pick data point $\vec{x}$
  - If sign($\vec{w}^T\vec{x}$) is correct class (1 or -1): do nothing
  - Otherwise: $\vec{w} = \vec{w} - \text{sign}(\vec{w}^T\vec{x})\vec{x}$

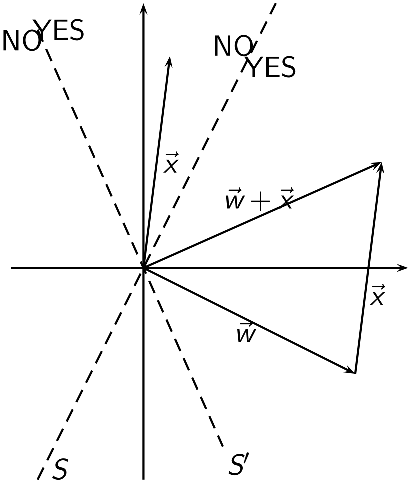
Perceptron (class of $\vec{x}$ is YES)



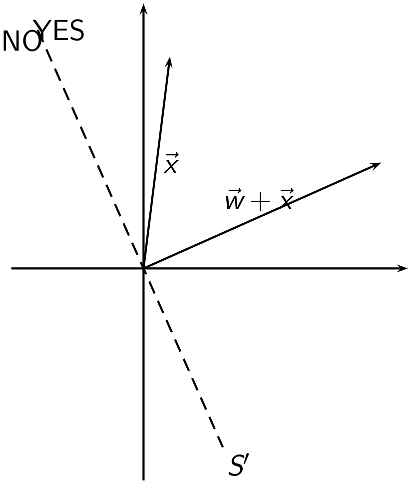
Perceptron (class of $\vec{x}$ is YES)



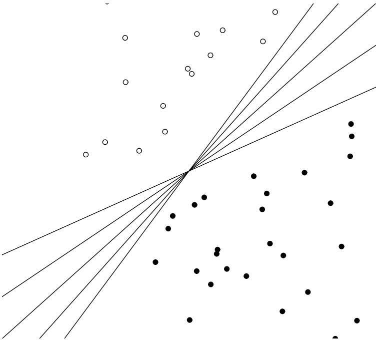
Perceptron (class of $\vec{x}$ is YES)



Perceptron (class of $\vec{x}$ is YES)



Which hyperplane?



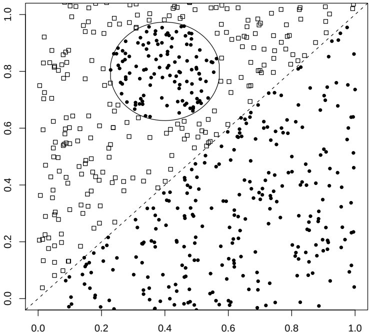
Which hyperplane?

- For linearly separable training sets: there are infinitely many separating hyperplanes.
- They all separate the training set perfectly ...
- ...but they behave differently on test data.
- Error rates on new data are low for some, high for others.
- How do we find a low-error separator?
- Perceptron: generally bad; Naive Bayes, Rocchio: ok; linear SVM: good

Linear classifiers: Discussion

- Many common text classifiers are linear classifiers: Naive Bayes, Rocchio, logistic regression, linear support vector machines etc.
- Each method has a different way of selecting the separating hyperplane
  - Huge differences in performance on test documents
- Can we get better performance with more powerful nonlinear classifiers?
- Not in general: A given amount of training data may suffice for estimating a linear boundary, but not for estimating a more complex nonlinear boundary.

A nonlinear problem

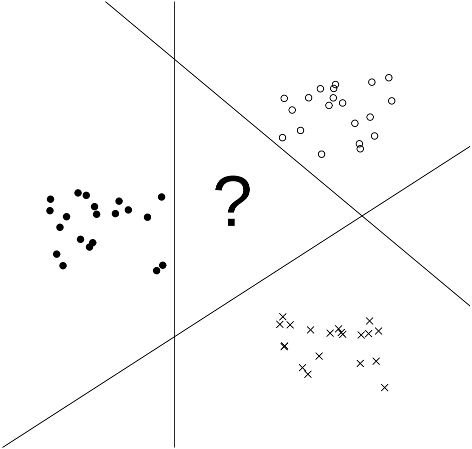


- Linear classifier like Rocchio does badly on this task.
- kNN will do well (assuming enough training data)

Which classifier do I use for a given TC problem?

- Is there a learning method that is optimal for all text classification problems?
- No, because there is a tradeoff between bias and variance.
- Factors to take into account:
  - How much training data is available?
  - How simple/complex is the problem? (linear vs. nonlinear decision boundary)
  - How noisy is the problem?
  - How stable is the problem over time?
    - For an unstable problem, it's better to use a simple and robust classifier.

How to combine hyperplanes for > 2 classes?



One-of problems

- One-of or multiclass classification
  - Classes are mutually exclusive.
  - Each document belongs to exactly one class.
  - Example: language of a document (assumption: no document contains multiple languages)

One-of classification with linear classifiers

- Combine two-class linear classifiers as follows for one-of classification:
  - Run each classifier separately
  - Rank classifiers (e.g., according to score)
  - Pick the class with the highest score

Any-of problems

- Any-of or multilabel classification
  - A document can be a member of 0, 1, or many classes.
  - A decision on one class leaves decisions open on all other classes.
  - A type of "independence" (but not statistical independence)
  - Example: topic classification
  - Usually: make decisions on the region, on the subject area, on the industry and so on "independently"

Any-of classification with linear classifiers

- Combine two-class linear classifiers as follows for any-of classification:
  - Simply run each two-class classifier separately on the test document and assign document accordingly