HMM: esercizi

Corso Deep Learning – a.a. 2024-25

HMM for a bit register

Consider a two-bit register. The register has four possible states: 00, 01, 10 and 11. Initially, at time 0, the contents of the register is chosen at random to be one of these four states, each with equal probability. At each time step, beginning at time 1, the register is randomly manipulated as follows: with probability 0.5, the register is left unchanged; with probability 0.25, the two bits of the register are exchanged (e.g., 01 becomes 10); and with probability 0.25, the right bit is flipped (e.g., 01 becomes 00). After the register has been manipulated in this fashion, the left bit is observed. Suppose that on the first three time steps, we observe 0, 0, 1.

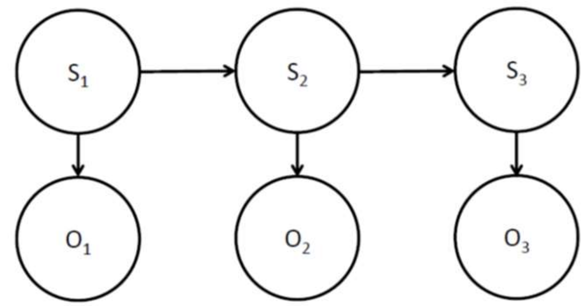
(a) Formulate the register as an HMM. What is the probability of transitioning from every state to every other state? What is the probability of observing each output (0 or 1) in each state?

(b) What is the most likely sequence of states given all three observed bits? (Be sure to include the initial state at time 0 in your sequence)

Solution

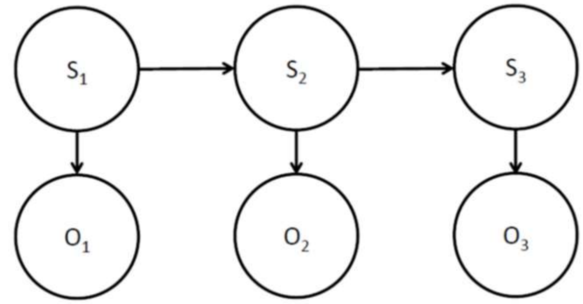
- Describe the HHM as a graph with *E*, *T* and as matrices/vectors
- Given the observation string "001" discuss the reply to (b) through a Trellis.
  - Describe (in pseudocode) the algorithm you followed to compute the temporary probabilities in the Trellis
  - Apply it to the string “1010” and determine the solution to the decoding problem

Ex 2:



Assume that we have the Hidden Markov Model (HMM) depicted in the figure above. If each of the states can take on k different values and a total of m different observations are possible (across all states), how many parameters are required to fully define this HMM? Justify your answer.

Ex 3:



State	$P(S_1)$
A	0.99
B	0.01

(a) Initial probs.

S_1	S_2	$P(S_2 \mid S_1)$
A	A	0.99
A	B	0.01
B	A	0.01
B	B	0.99

(b) Transition probs.

S	O	$P(O \mid S)$
A	0	0.8
A	1	0.2
B	0	0.1
B	1	0.9

(c) Emission probs.

Suppose that we have binary states (labeled A and B) and binary observations (labeled 0 and 1) and the initial, transition, and emission probabilities as in the given table.

3. Using the forward algorithm, compute the probability that we observe the sequence $O_1 = 0$, $O_2 = 1$, and $O_3 = 0$. Show your work (i.e., show each of your alphas).

SOLUTIONS

- Tbd during lesson