

The background of the slide features a faint, light gray pattern of interconnected nodes and lines, resembling a neural network or a complex web. A solid dark red vertical bar is positioned on the left side of the slide.

# Novel Trends: Low Rank, RAGs.

Roberto Basili, Danilo Croce  
Deep Learning, 2024/2025

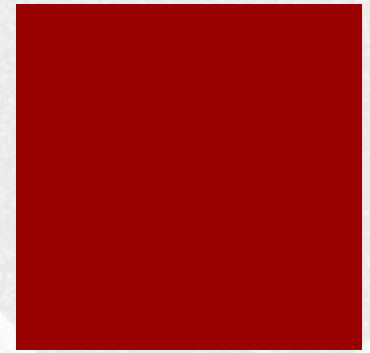
# Outline

- How to fine tune Large Scale Decoder-only architectures
  - Scale problems
  - Adapters for LLMs
- Alignment through External Sources
- Retrieval Augmented LLMs
  - RAG: the architecture
- Applications of RAGs
  - Vector Databases
  - Knowledge Distillation



How to train a large  
scale encoder?

# Challenge: 16GB GPU resources



ChatGPT's resources: 10-30.000 GPUs

1xTesla T4



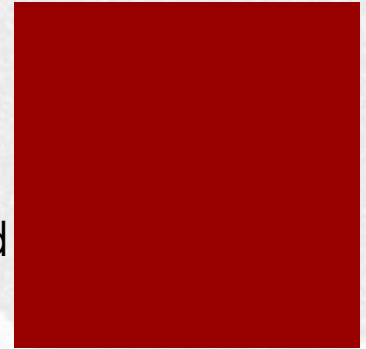
Render Config	
Shading Units:	2560
TMUs:	160
ROPs:	64
SM Count:	40
Tensor Cores:	320
RT Cores:	40
L1 Cache:	64 KB (per SM)
L2 Cache:	4 MB

Theoretical Performance	
Pixel Rate:	101.8 GPixel/s
Texture Rate:	254.4 GTexel/s
FP16 (half):	65.13 TFLOPS (8:1)
FP32 (float):	8.141 TFLOPS
FP64 (double):	254.4 GFLOPS (1:32)

# Scale: impact



ChatGPT uses 10000 graphic cards and 285000 processor chips to process the data.



- How to **cool** them?
- Currently, to cool the systems used for GPT, **water is being used by Microsoft and OpenAI** (the method of cooling is called "evaporative cooling"). As reported in the research paper [1], Microsoft's state-of-the-art data center in the US can easily consume 700,000 litres of clean fresh water (potable water).
- To compare the metrics of water usage, the same amount of water can be used to
  - manufacture 370 BMW cars
  - manufacture 320 Tesla Evs
  - could quench the thirst of 2,30,000 people (considering an avg of 3 litres of water drunk in a day by a person) in one single day

# Pretrainable Adapters: the idea

$$h_x^{(A)} = \text{MLP}_u(g_2(\text{Enc}(g_1(\text{MLP}_d(h_x)))). \quad (1)$$

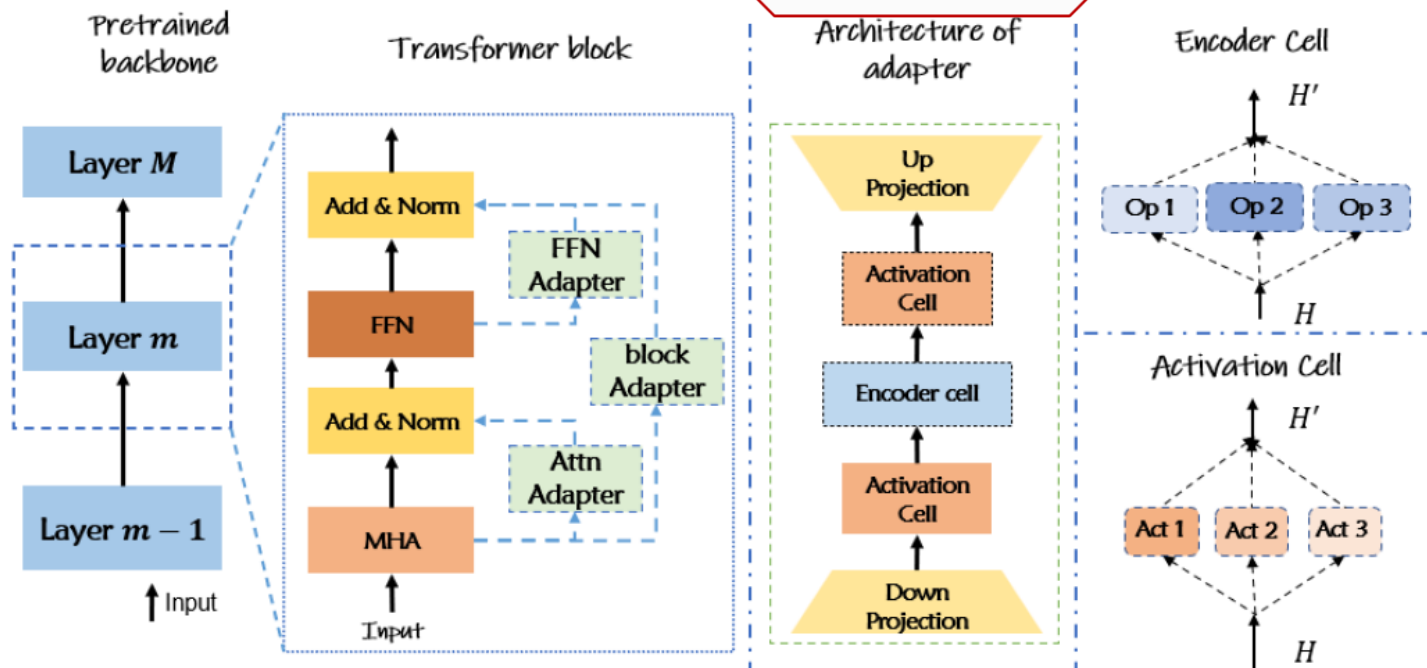


Figure 2: The overall framework of our Learned Adapter.

*What are the optimal architectures for adapters?*

# Architecture Search methods

- Architectural Choices:
  - Which Activation function?
  - Which Encoder?
  - Which Adapter Placement?
- DART algorithm for architecture search
  - Define the search options  $O$  and design an *hypernetwork* where layer weights and architectural parameters  $O$  are trainable in an end-to-end fashion
  - Extracts the final sub-network a posteriori by selecting the best operation on each edge and dropping the lower-score operations
  - Retrain from scratch the final sub-network on the original train set with randomly initialized parameters.

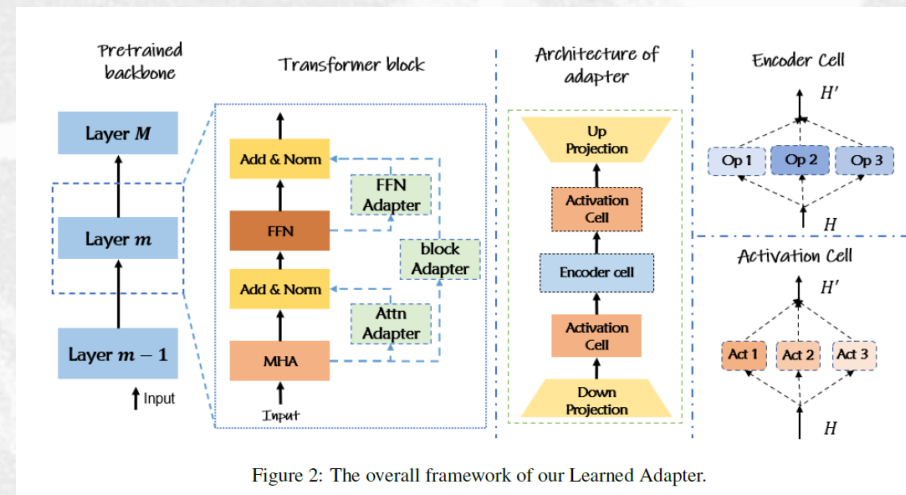


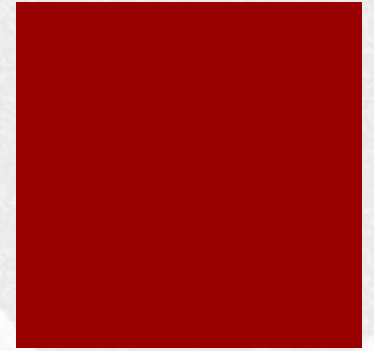
Figure 2: The overall framework of our Learned Adapter.

# Architecture Search: Outcomes

Layer index	Adapter placement	Activation $g_1$	Activation $g_2$	Encoder operation 1
1	FFN	elu	null_act	mha_8
2	-	-	-	-
3	-	-	-	-
4	FFN	elu	null_act	mha_2
5	Block	tanh	null_act	mha_2
6	FFN	gelu_new	leaky_relu	conv_3
7	FFN	null_act	tanh	mha_2
8	-	-	-	-
9	Attn	elu	relu	conv_3
10	-	-	-	-
11	Attn	gelu_new	relu	conv_1
12	Attn	gelu_new	relu	conv_3
13	Block	relu	leaky_relu	conv_1
14	Attn	swish	relu	conv_3
15	FFN	leaky_relu	relu	conv_3
16	Block	leaky_relu	relu	conv_5
17	FFN	leaky_relu	relu	conv_1
18	Attn	leaky_relu	null_act	skip_connect
19	FFN	relu	relu	conv_3
20	FFN	gelu_new	null_act	conv_3
21	-	-	-	-
22	Block	tanh	null_act	mha_8
23	Attn	tanh	null_act	conv_3
24	FFN	tanh	null_act	mha_2

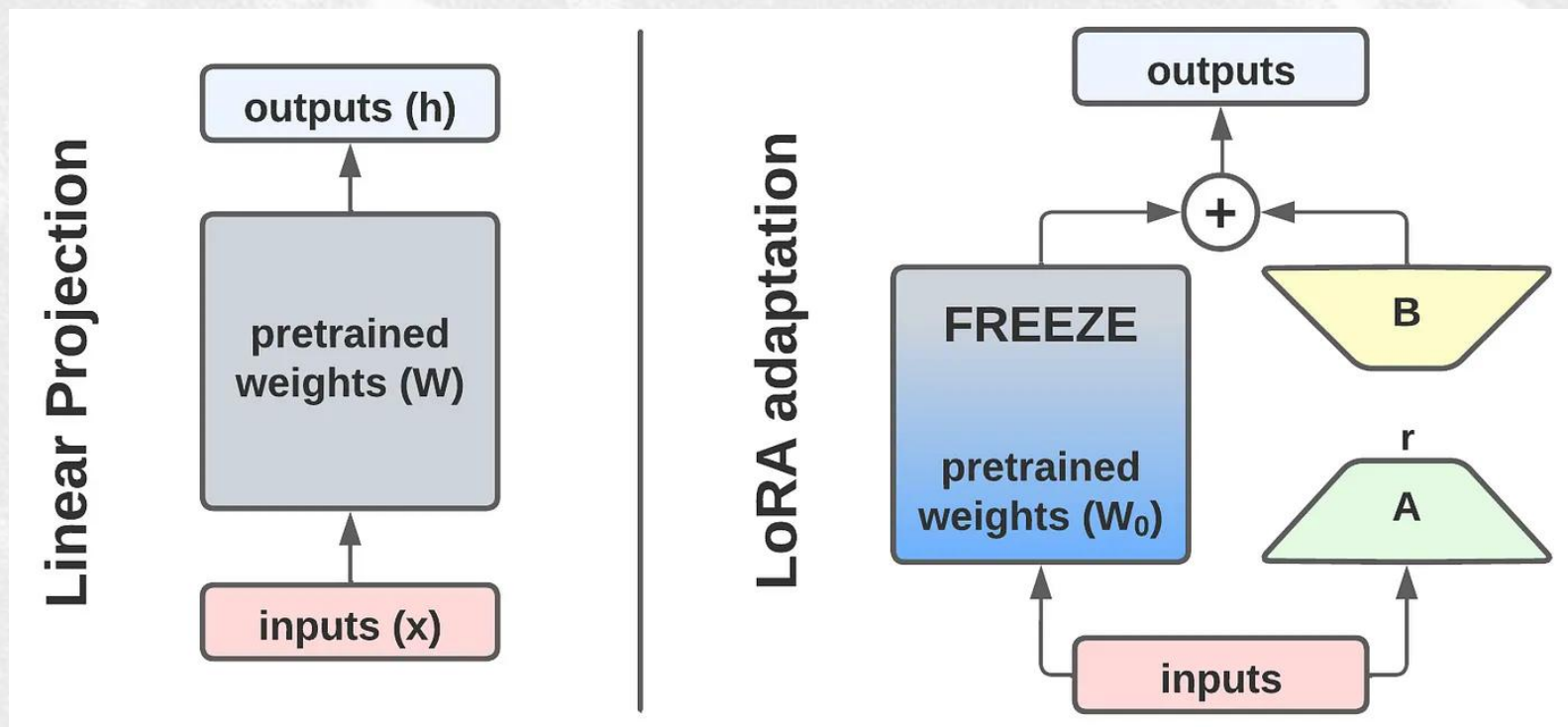
Table 9: The learned adapter architectures on the RTE task when the PTM backbone is RoBERTa-large. If an adapter’s architecture contains only “-”, it means our Learned Adapter framework choose the **null** encoder operation, and equivalently, dropping this layer’s adapter.

# Low Rank Adaptation: Motivations

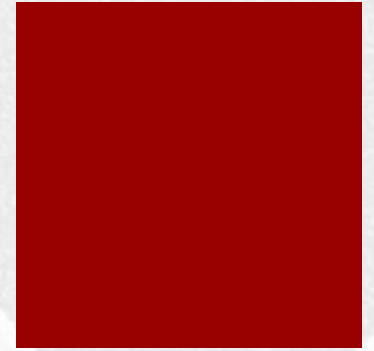


- Fine-tuning is computationally challenging when applied to large pre-trained models as it involves the **adjustment of millions of parameters**. Effective traditional fine-tuning demands huge computational resources and time, so that it has a limited applicability to model adaption for specific tasks.
- In traditional fine-tuning, the adjustment involves altering the original weight matrix  $\mathbf{W}$  of the network. The changes made to  $\mathbf{W}$  during fine-tuning can be collectively represented by  $\Delta\mathbf{W}$ , such that the updated weights can be expressed as  $\mathbf{W} + \Delta\mathbf{W}$
- As *intrinsic rank hypothesis* may suggest, the significant changes (i.e.  $\Delta\mathbf{W}$ ) to the neural network can be captured just relying on a small lower-dimensional representation.

# Adapters: the idea

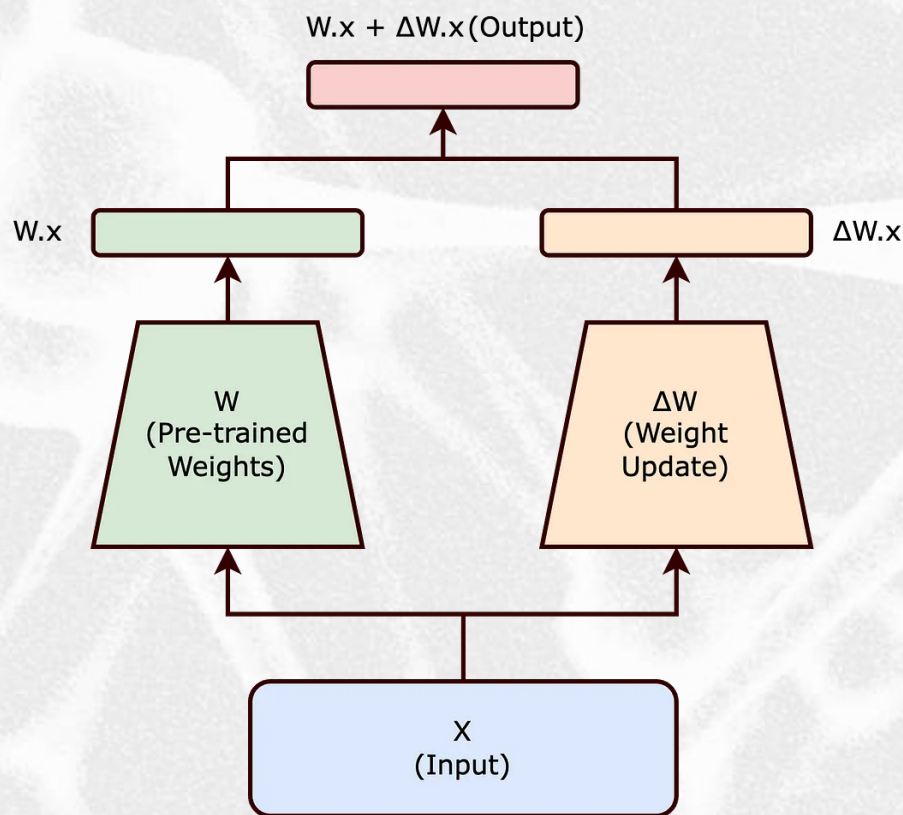


# LoRA: aims

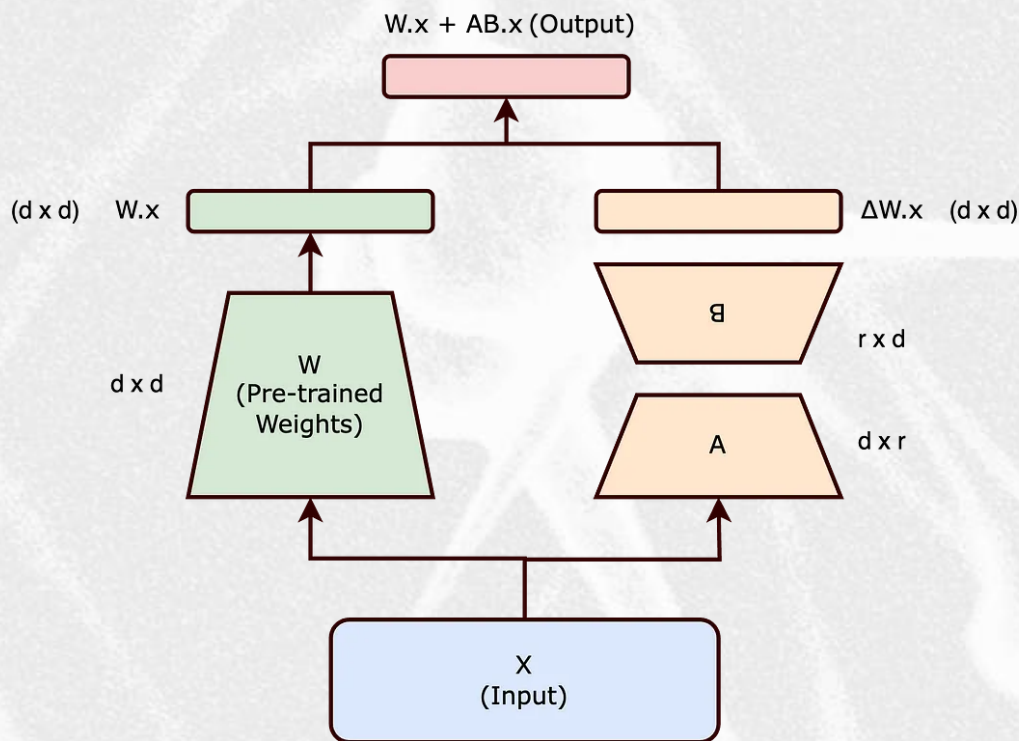


- A pre-trained model can be shared and used to build many small LoRA modules for different tasks.
  - The shared model can be freezed and efficient switching among tasks is achieved by replacing the matrices A and B, reducing the storage requirement and task-switching overhead significantly.
- LoRA makes training more efficient and lowers the hardware barrier to entry by up to 3 times when using adaptive optimizers since we do not need to calculate the gradients or maintain the optimizer states for most parameters.
  - Only the injected, much smaller low-rank matrices are optimized.
- A simple linear design allows us to merge the trainable matrices with the frozen weights when deployed, that does not introduce any inference latency compared to a fully fine-tuned model, by construction.
- LoRA is orthogonal to many prior fine-tuning methods and can be combined with many of them, such as prefix-tuning.

# Exploiting implicit Low Rank



# Exploiting implicit Low Rank (2)



Low Rank of  $A$  and  $B$  implies a rank  $r$  (with  $r \ll d$ ) significantly reduces the number of trainable parameters.

If  $W$  is a  $d \times d$  matrix, standard  $W$  updating involves  $d^2$  parameters.

With  $B$  and  $A$  of sizes  $d \times r$  and  $r \times d$  respectively, the total number of parameters reduces to  $2dr$ , which is much smaller when  $r \ll d$ .

# Low-Rank Adaptation (LoRA)

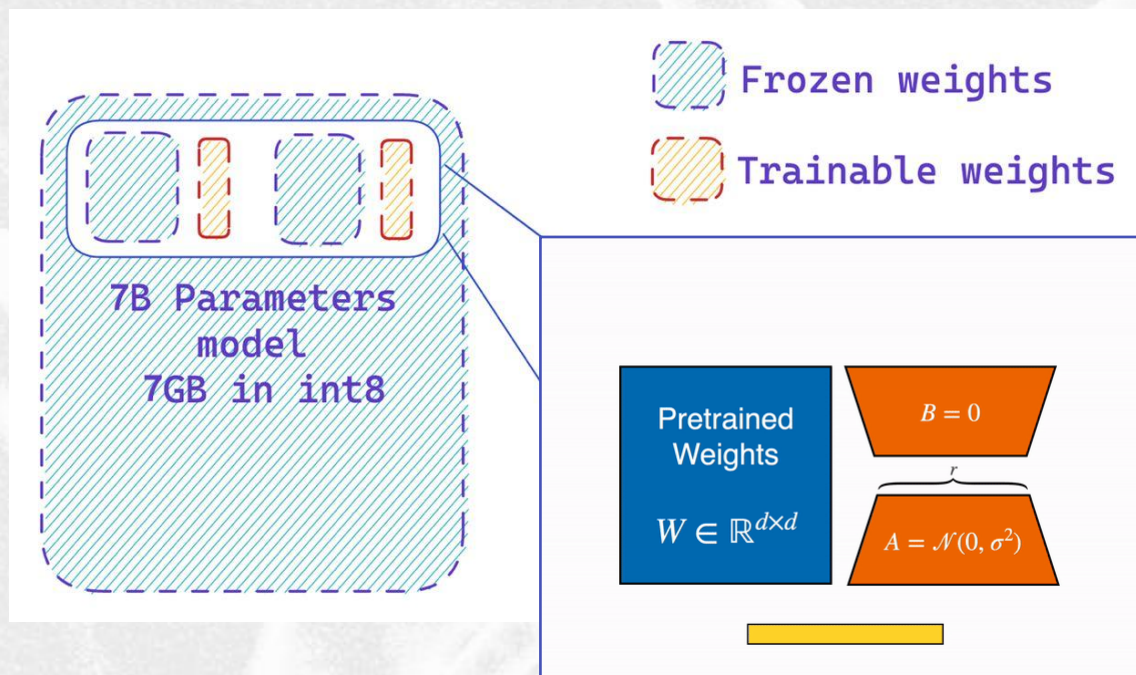
(Hu et al., 2021)

Low Rank Adaptation (LoRA: Hu et al., 2021): create the parallel (fine-tunable) adapters as smaller matrices:

- add the adapters to the base model while keeping the base model frozen

LoRA is **NOT** learning any parameter, but the **changes in the parameters!**

$$\underbrace{W_0 + \Delta W}_{\text{Traditional FT}} = \underbrace{W_0}_{\text{LoRA FT}} + \underbrace{BA}_{\text{LoRA FT}}$$



# Advantages

- When applied to very large language models, the Low-Rank Adaptation (LoRA) method largely reduces the number of trainable parameters, offering several benefits, :
  1. **Reduced Memory Requirements:** LoRA decreases memory needs by lowering the number of parameters to update, aiding in the management of large-scale models.
  2. **Faster Adaptation/Training:** By simplifying computational demands, LoRA accelerates the training and fine-tuning of large models for new tasks.
  3. **Lower HW requirements:** LoRA's lower parameter count enables the fine-tuning of substantial models on less powerful hardware, like modest GPUs or CPUs.
  4. **Larger Scale Models:** LoRA facilitates the expansion of AI models without a corresponding increase in computational resources, making the management of growing model sizes more practical.

# LoRA trends: ALoRA

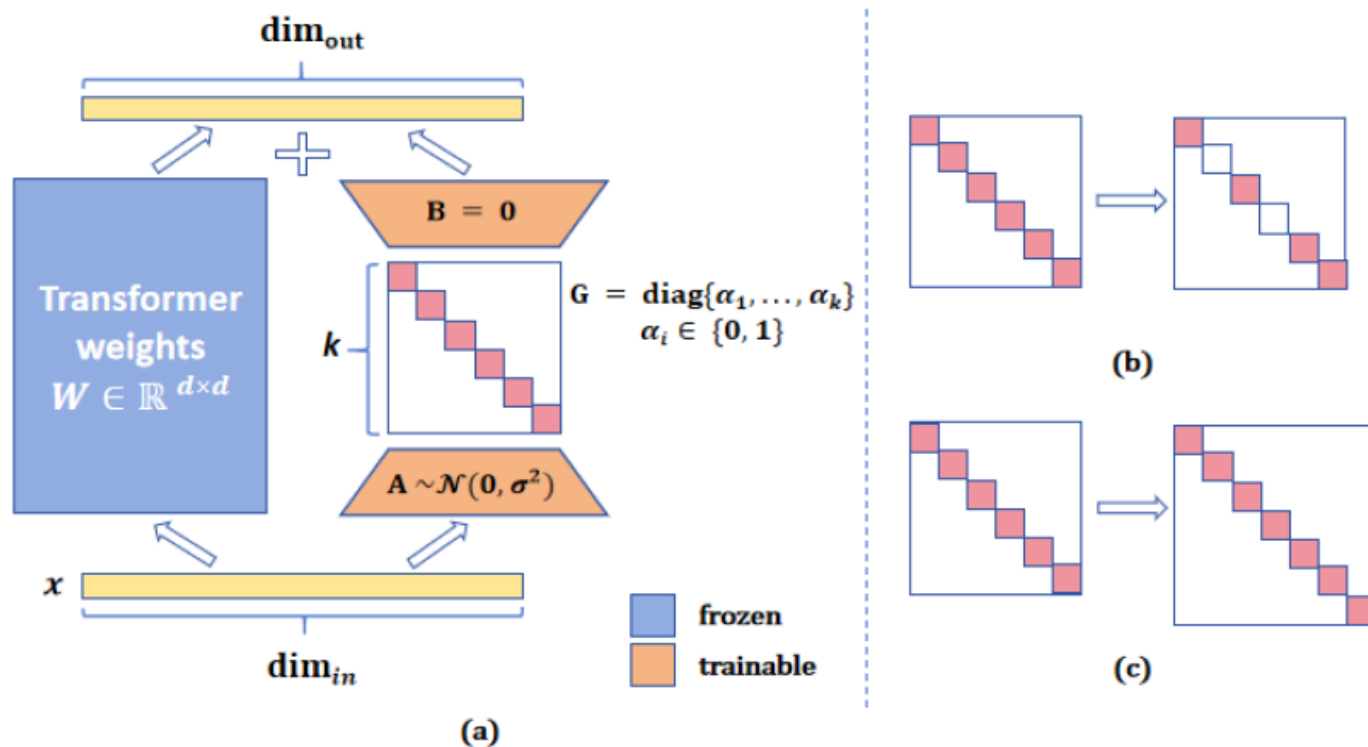


Figure 1: Schematic illustration of our ALoRA. Left (a): ALoRA follows LoRA to update the weight matrix  $W$  by fine-tuning the low-rank matrices  $A$  and  $B$  with intermediate rank  $k$ . Matrix  $G$  is a diagonal matrix where each diagonal element is the gate unit  $\alpha_i$  for each LoRA rank  $i < k$ . Each  $\alpha_i$  is set to 1 at initialization. Right upper (b): Some abundant LoRA ranks are pruned by setting the corresponding gate  $\alpha_i$  to zeros. Right lower (c): For weight matrix  $W$  whose LoRA ranks are not pruned, we will assign additional LoRA ranks to enhance reparameterization.

# ALoRA: algorithm

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## Algorithm 1: Workflow of ALoRA

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**Input:** A super-network  $M$ , with  $R^{target}$  LoRA ranks uniformly distributed in modules of  $M$ ;

**Output:** A new allocation of  $R^{target}$  LoRA ranks.

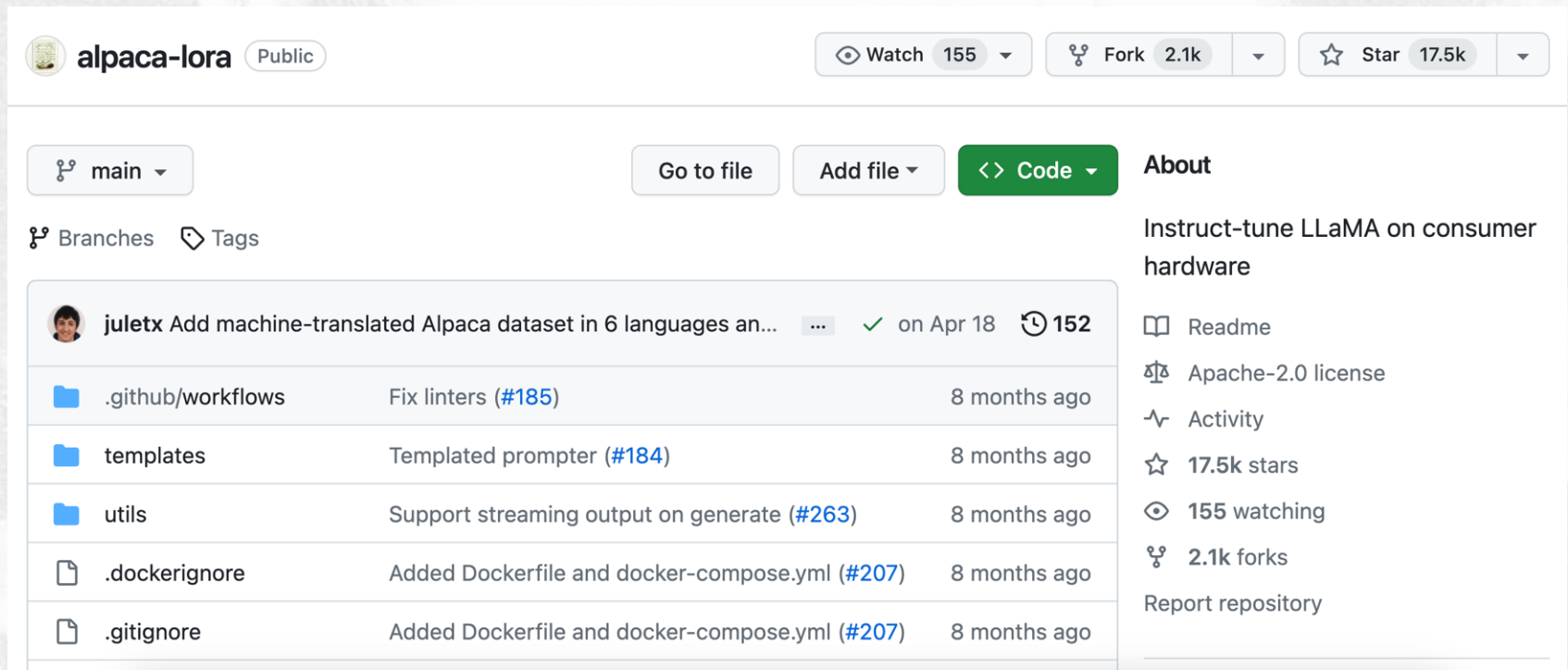
**Data:** Training set  $D_{train}$ , a batch of validation data  $B_{val}$

- 1 Train super-network  $M$  on the training set  $D_{train}$  for  $K_1$  epochs;
  - 2 **for**  $n = 0$ ;  $n < N_A$  **do**
  - 3     **for** a single LoRA rank  $r_{m,i}$  on  $M$  **do**
  - 4         Calculate the importance score  $IS(r_{m,i})$  on  $B_{val}$ ;
  - 5     Prune  $n_0$  LoRA ranks with lowest importance scores;
  - 6     **if** there are modules not pruned **then**
  - 7         Add  $n_0$  LoRA ranks to the un-pruned modules;
  - 8     Further train the Super-network  $M$  on  $D_{train}$  for  $K_2$  epochs;
-

# Alpaca LoRA

Within just a few days following the release of Alpaca's training material, LoRA was utilized to fine-tune LLaMa into Alpaca efficiently, using only a «small» GPU:

- <https://github.com/tloen/alpaca-lora>



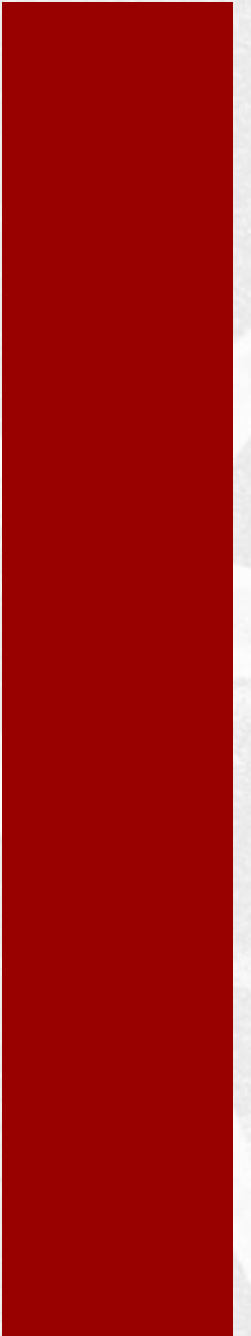
The screenshot shows the GitHub repository page for **alpaca-lora** by user **tloen**. The repository is public and has 155 watchers, 2.1k forks, and 17.5k stars. The main branch is **main**. The repository description is "Instruct-tune LLaMA on consumer hardware". The repository includes a README, Apache-2.0 license, and activity feed. The file list shows the following structure:

File/Folder	Description	Commit Count	Time Ago
<b>.github/workflows</b>	Fix linters	#185	8 months ago
<b>templates</b>	Templated prompter	#184	8 months ago
<b>utils</b>	Support streaming output on generate	#263	8 months ago
<b>.dockerignore</b>	Added Dockerfile and docker-compose.yml	#207	8 months ago
<b>.gitignore</b>	Added Dockerfile and docker-compose.yml	#207	8 months ago

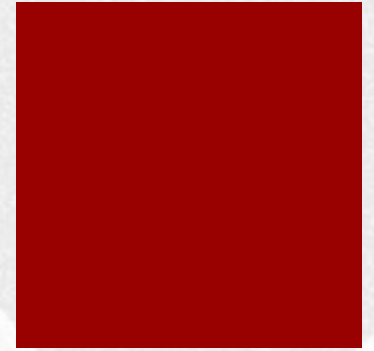
On the right side, there is a sidebar with the following information:

- About**
- Instruct-tune LLaMA on consumer hardware
- Readme
- Apache-2.0 license
- Activity
- 17.5k stars
- 155 watching
- 2.1k forks
- Report repository

# Aligning LLMs



# RAG: motivations



- Large pre-trained language models have been shown to **store factual knowledge** in their parameters, and **achieve state-of-the-art results** when fine-tuned on downstream NLP tasks.
- However, **their ability to access and precisely manipulate knowledge is still limited**, and hence on knowledge-intensive tasks, their performance lags behind task-specific architectures.
- Additionally, **providing provenance for their decisions** and **updating their world knowledge** remain open research problems.

# Knowledge Integration and LLMs: RAG Models

- Retrieval Augmented Generation (Lewis et al., 2020)
- At *generation time* contextual information able to qualify the LLM response is made available
- It is essential for **knowledge intensive** tasks
- It is possible to apply RAG either to the *pre-training* or to the *fine-tuning* and *prompting* stage
- It has been shown to impact positively onto hallucinations

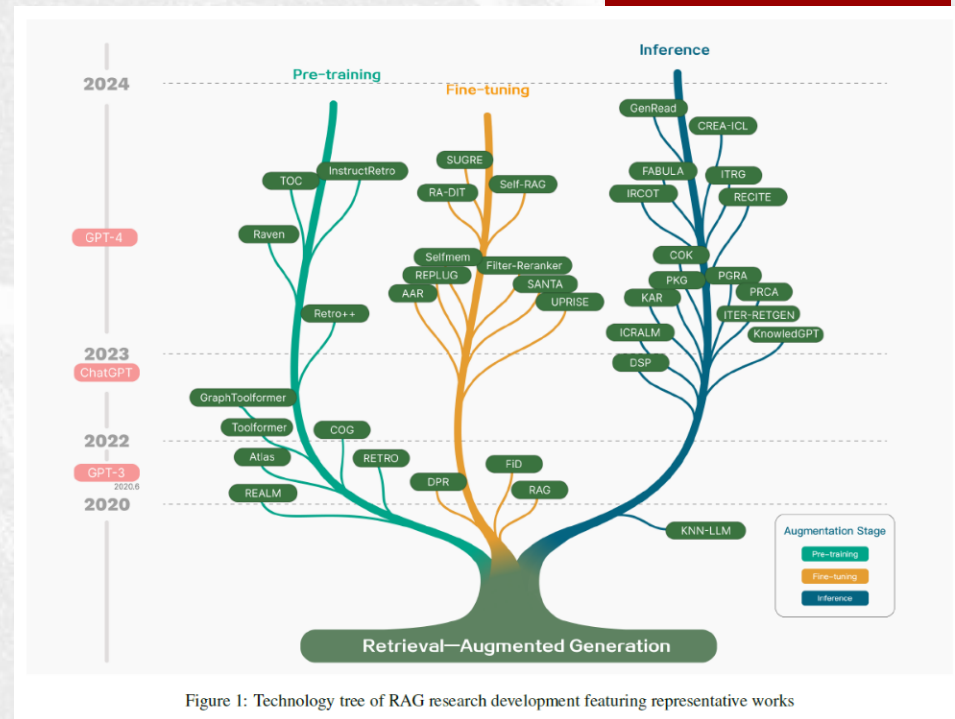
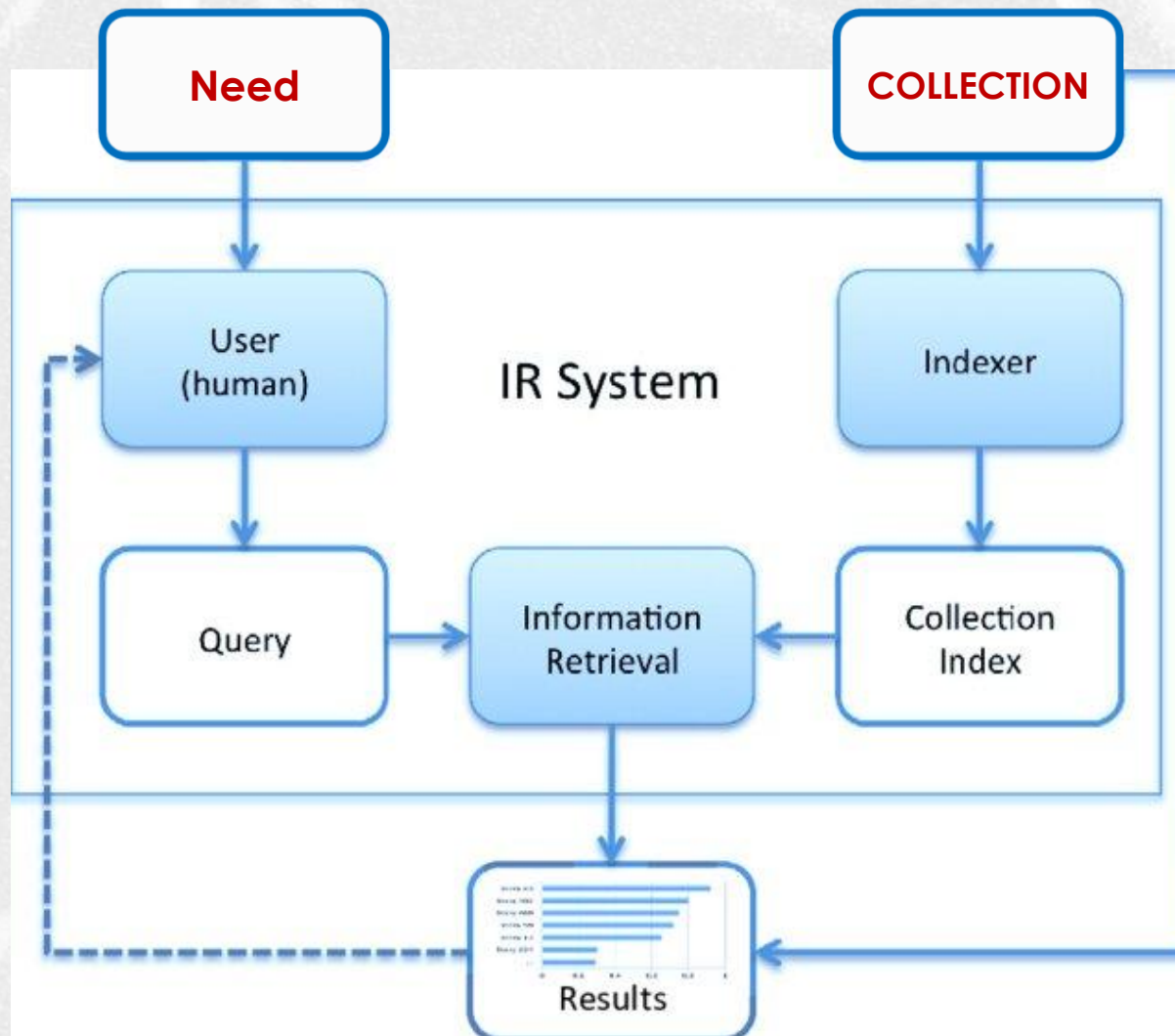


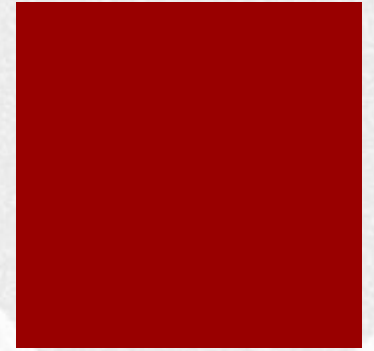
Figure 1: Technology tree of RAG research development featuring representative works

(Lewis et al, 2020) [Retrieval-augmented generation for knowledge-intensive NLP tasks. Proceedings of NIPS, Advances in Neural Information Processing Systems, 33 \(2020\): 9459-9474.](#)

# The basic Retrieval workflow

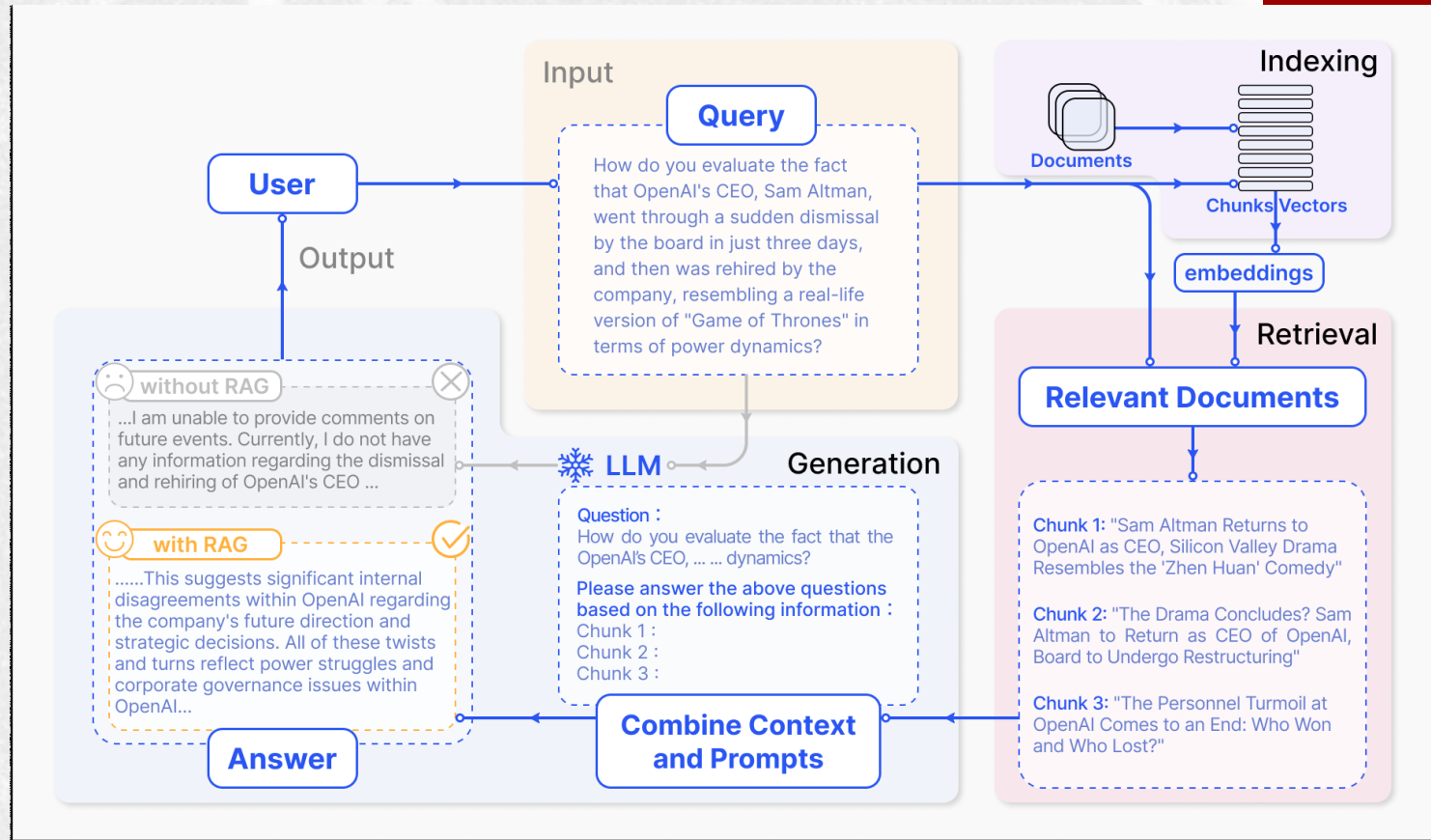


# RAG: the steps



1. INPUT: It corresponds to the question posed to an LLM system. If no RAG is applied, LLM responds to the question through standard decoding
2. INDEXING: To employ RAG, a set of reference documents are to be indexed.
  - It involves **chunking** the documents, **embeddings these chunks**, and then **indexing embeddings** into a *vector store*.
  - The **input query is also embedded**.
3. RETRIEVAL: Relevant documents are retrieved by **comparing the query embedding against the document vectors**.
4. GENERATION: Retrieved documents are first merged with the original prompt to provide additional context and then the LLM response generation is triggered:
  - This combined text and prompt is the input for response generation, that produces the final output provided to the user.

# RAG models: the information flow



# RAG models: the training task

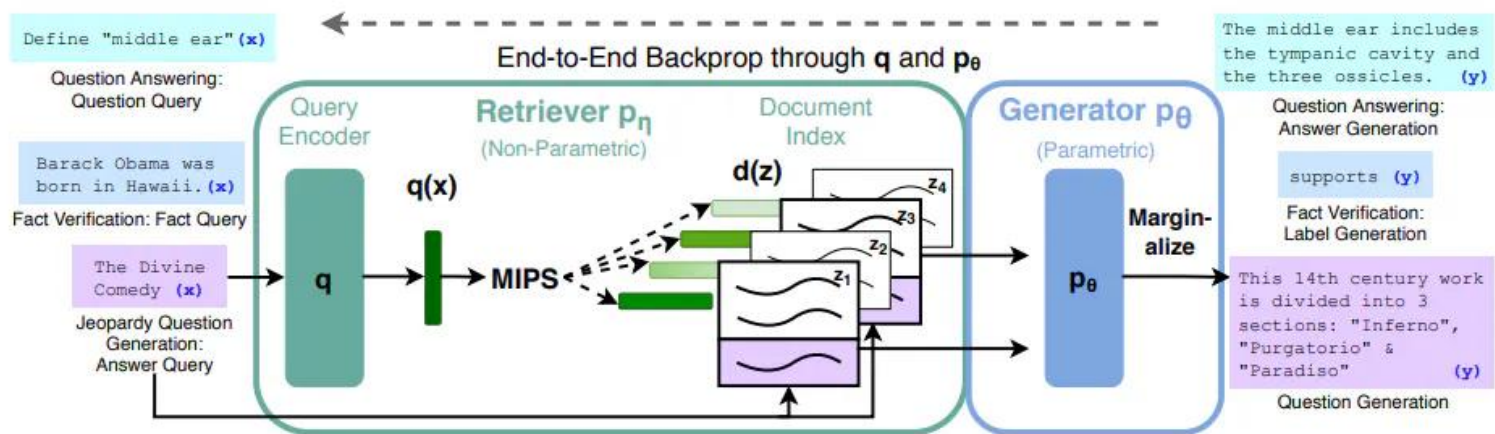
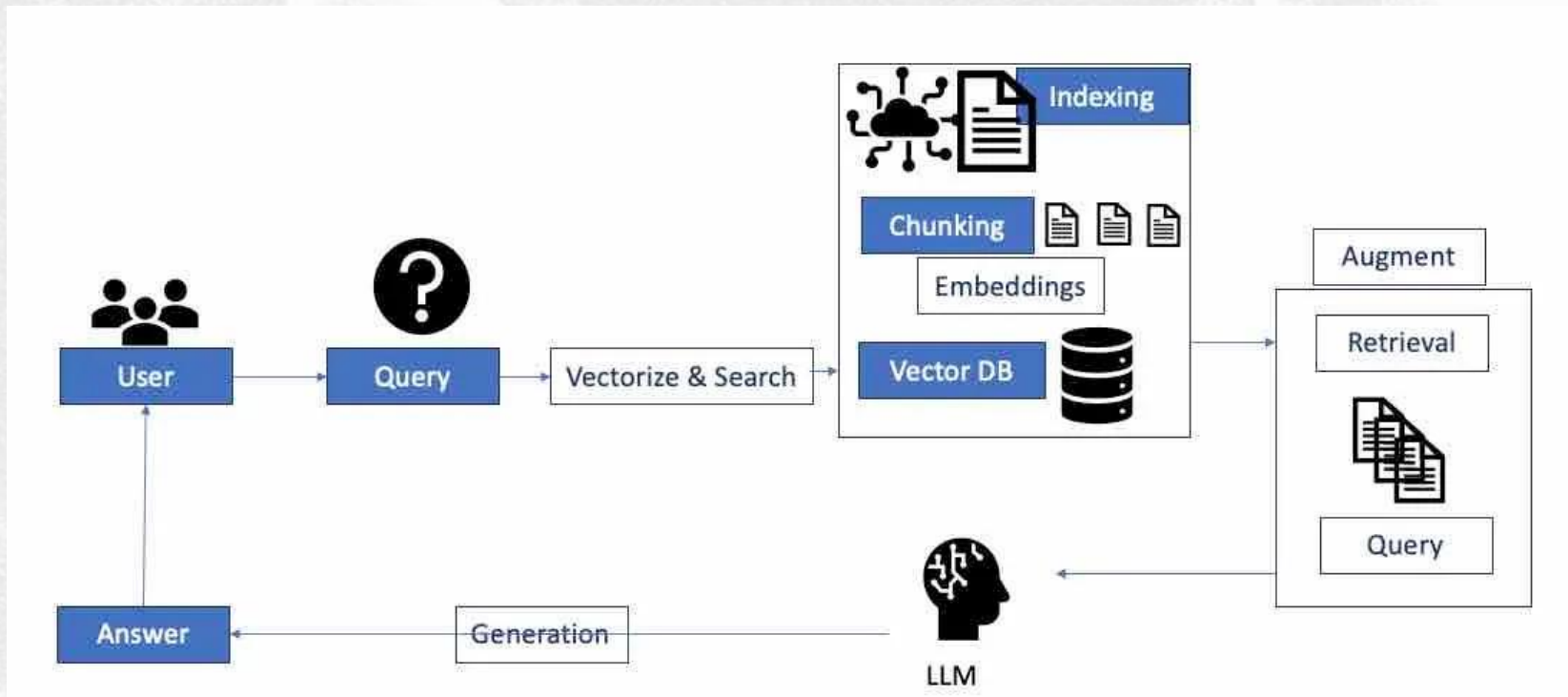
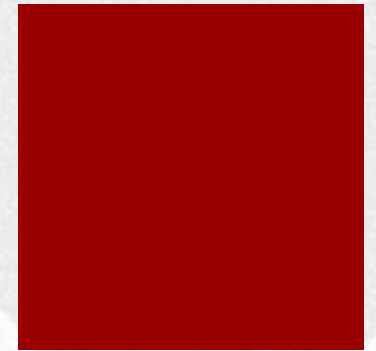
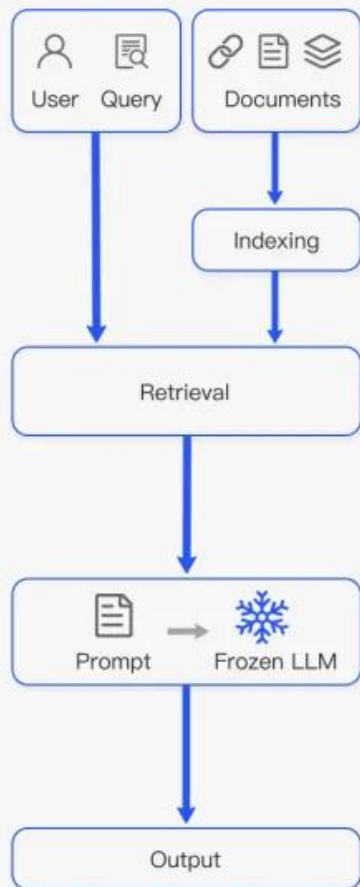
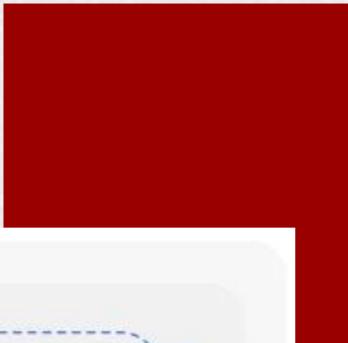


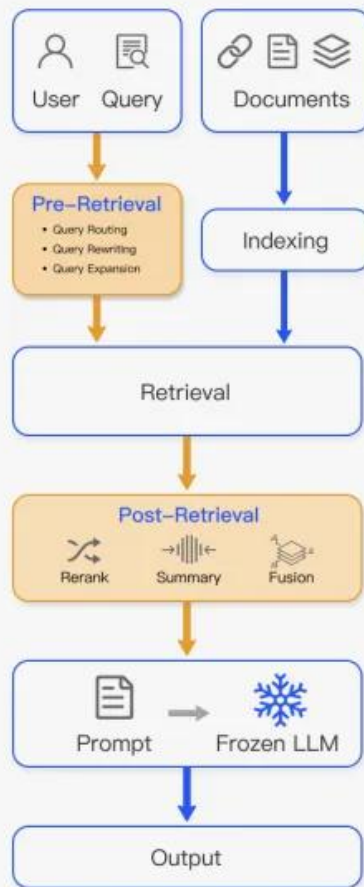
Figure 1: Overview of our approach. We combine a pre-trained retriever (*Query Encoder + Document Index*) with a pre-trained seq2seq model (*Generator*) and fine-tune end-to-end. For query  $x$ , we use Maximum Inner Product Search (MIPS) to find the top-K documents  $z_i$ . For final prediction  $y$ , we treat  $z$  as a latent variable and marginalize over seq2seq predictions given different documents.

# The RAG architecture

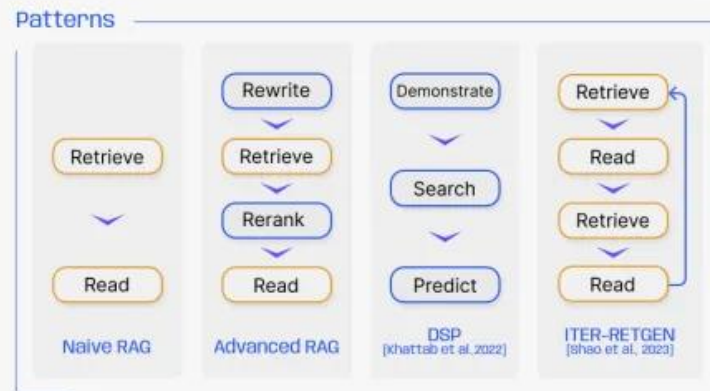
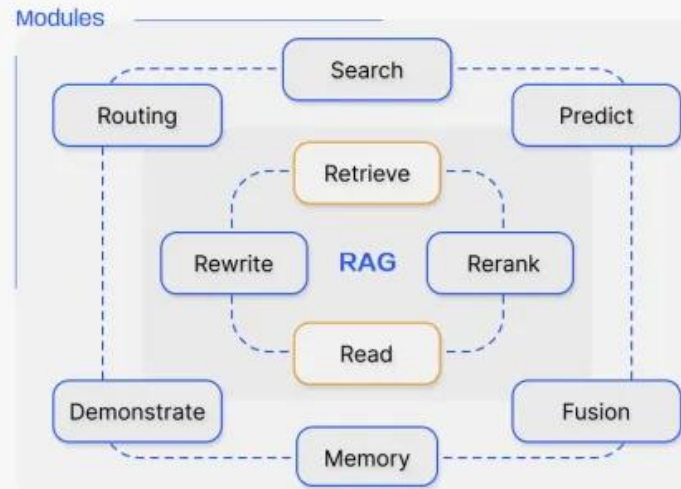




## Naive RAG



## Advanced RAG



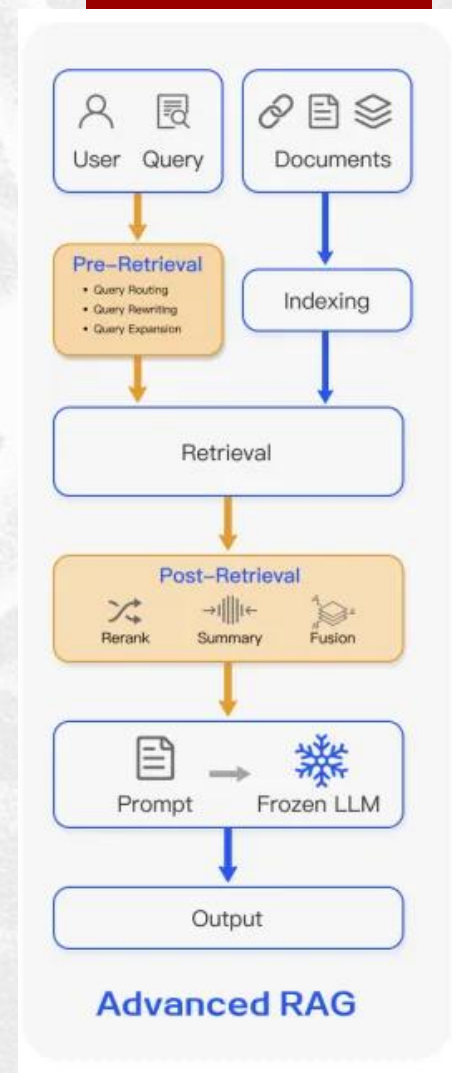
## Modular RAG

# Advanced RAGs

- It employs **optimization** across the (A) **pre-retrieval**, (B) **retrieval**, and (C) **post-retrieval** processes.

A. The **pre-retrieval phase** involves refining data indexing through five key stages:

- enhancing data granularity,
- optimizing index structures,
- adding metadata,
- alignment optimization, and
- mixed retrieval

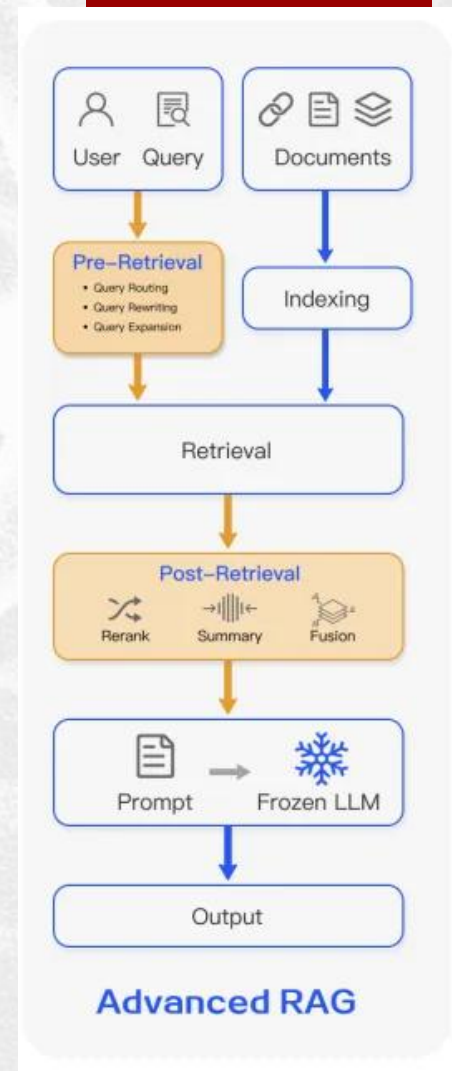


# Advanced RAGs

- It employs **optimization** across the (A) **pre-retrieval**, (B) **retrieval**, and (C) **post-retrieval** processes.

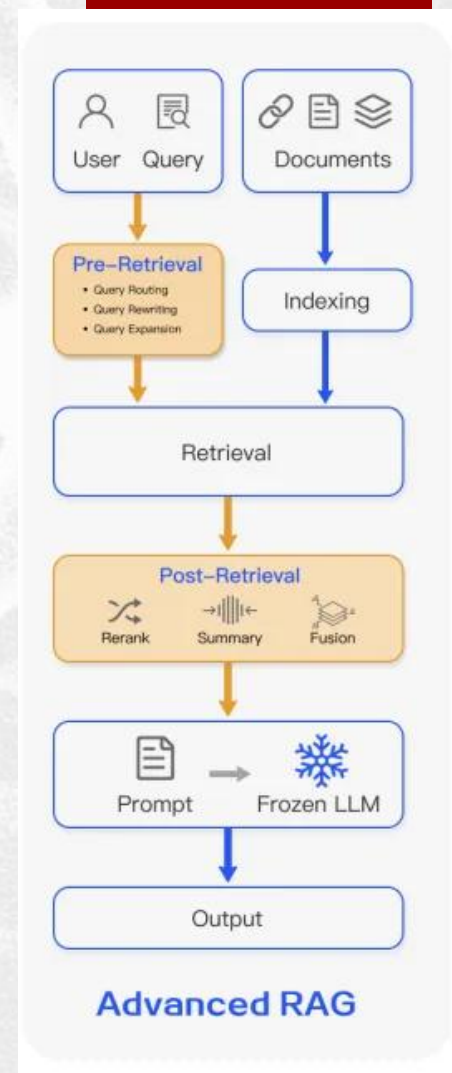
B. The **retrieval phase involves optimizing the embedding model itself** to maximize the quality of context chunks. Strategies may include:

- **fine-tuning embeddings** to improve retrieval relevance or
- **employing dynamic embeddings** that better capture contextual nuances (e.g., OpenAI's embeddings-ada-02 model)



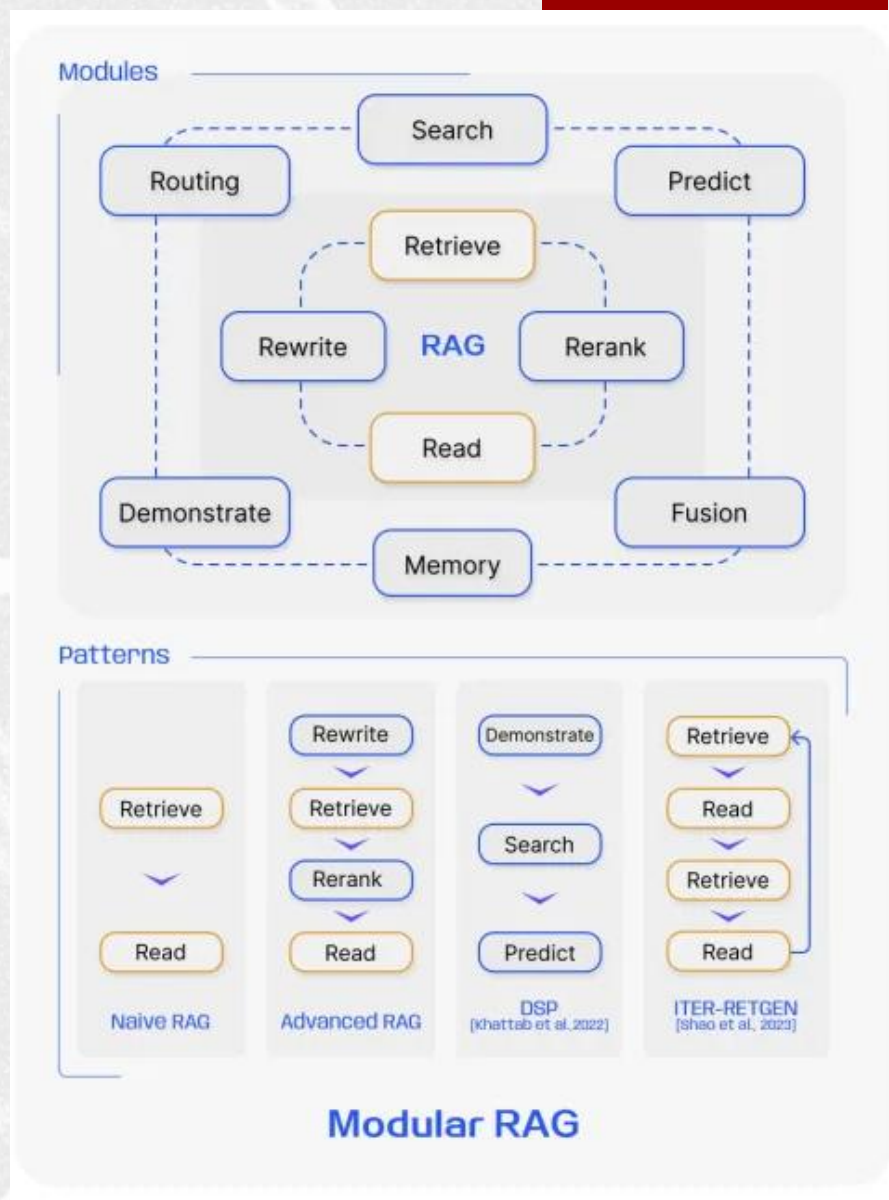
# Advanced RAGs

- It employs **optimization** across the (A) **pre-retrieval**, (B) **retrieval**, and (C) **post-retrieval** processes.
- C. The **post-retrieval phase** focuses on **circumventing context window limitations** and **managing noisy or distracting information**. **Re-ranking is a common approach** to address these challenges, involving techniques such as
  - **relocating relevant context to the edges of the prompt** or
  - **recalculating semantic similarity between the query and relevant text chunks**.
  - **Prompt compression techniques** may also aid

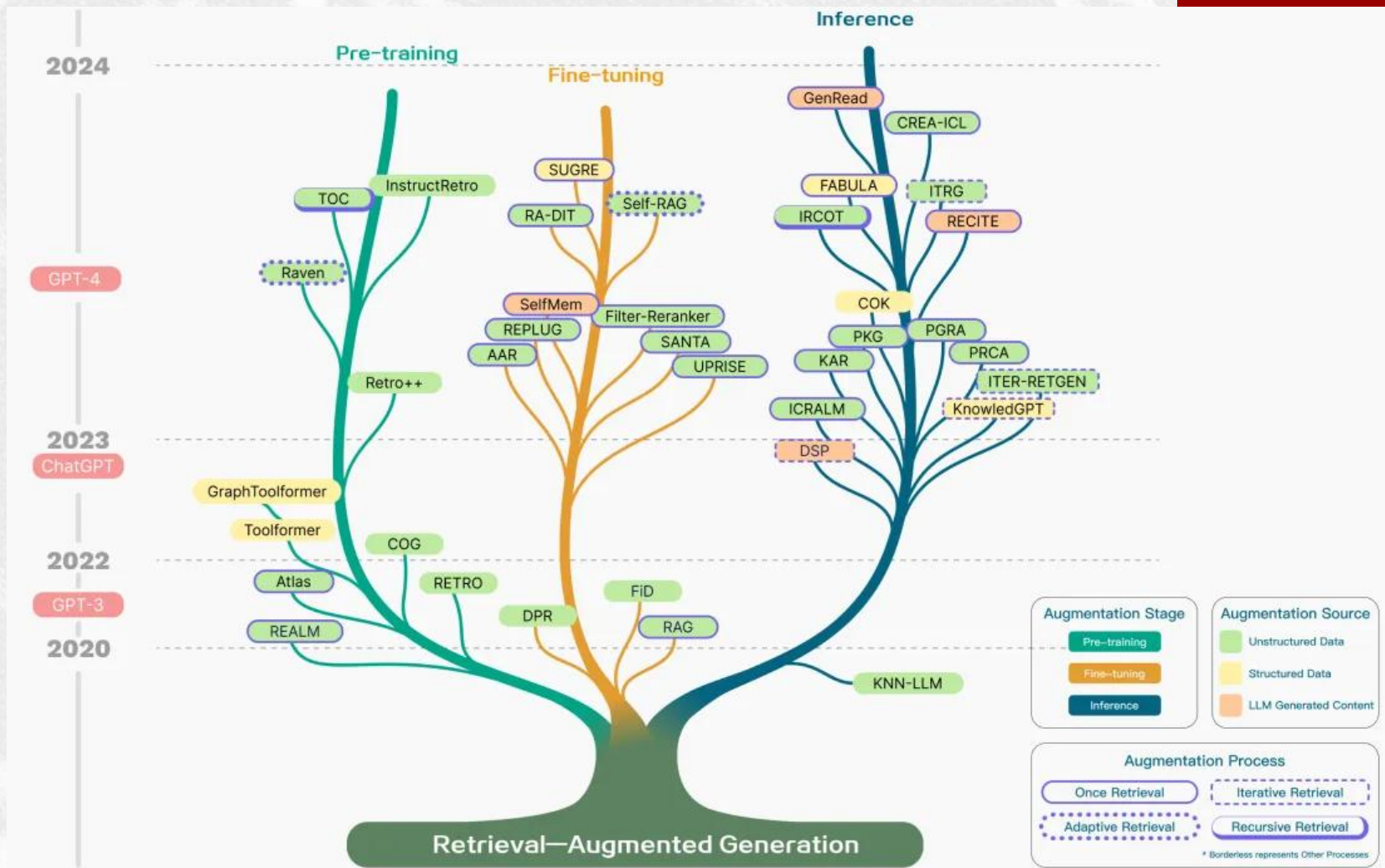


# Modular RAG

- **SEARCH MODULE:** Tailored for specific use-cases, it can perform direct searches on various corpora, utilizing LLM-generated code and query languages.
- **MEMORY MODULE:** Uses the LLM's memory for retrieval, improving alignment with data distributions.
- **FUSION:** Expands user queries into diverse perspectives, improving search results through multi-query approaches and re-ranking.
- **ROUTING:** Determines actions for queries, selecting the appropriate data source for retrieval.
- **PREDICT:** Uses the LLM to generate context instead of direct retrieval to reduce redundancy and noise.
- **TASK ADAPTER:** Adapts RAG to various tasks, enhancing universality and creating task-specific retrievers.



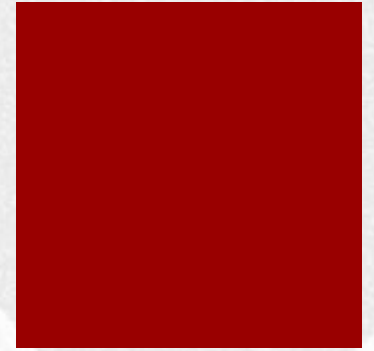
# The foundational RAGs



# RAG evaluation

- The evaluation of a RAG framework focuses on **three primary quality scores and four abilities**.
- **QUALITY SCORES** encompass measuring
  - **context relevance** (precision and specificity of retrieved context),
  - **answer faithfulness** (faithfulness of answers to retrieved context), and
  - **answer relevance** (relevance of answers to posed questions).
- Additionally, four abilities measure **ADAPTABILITY AND EFFICIENCY** of a RAG system:
  - **noise robustness**,
  - **negative rejection**,
  - **information integration**, and
  - **counterfactual robustness**.

# RAG evaluation: DEFs



- Context Relevance - Precision and Specificity of retrieved context (*How much does the context actually relate to the query?*)
- Answer Faithfulness - *Is the answer true to the retrieved context? Is it making anything up that isn't within the context?*
- Answer Relevance - *Is the answer actually relevant to the core meaning of the query?*
- Noise Robustness - *How well can the model ignore useless information that is retrieved?*
- Negative Rejection - *How well can the model refrain from responding when the context does not have the necessary information included?*
- Information Integration - *How well can the model combine all of the information into a clean and summarized answer?*
- Counterfactual Robustness - *How well can the model recognize that the provided context is actually wrong, and discard the information?*

# RAG evaluation

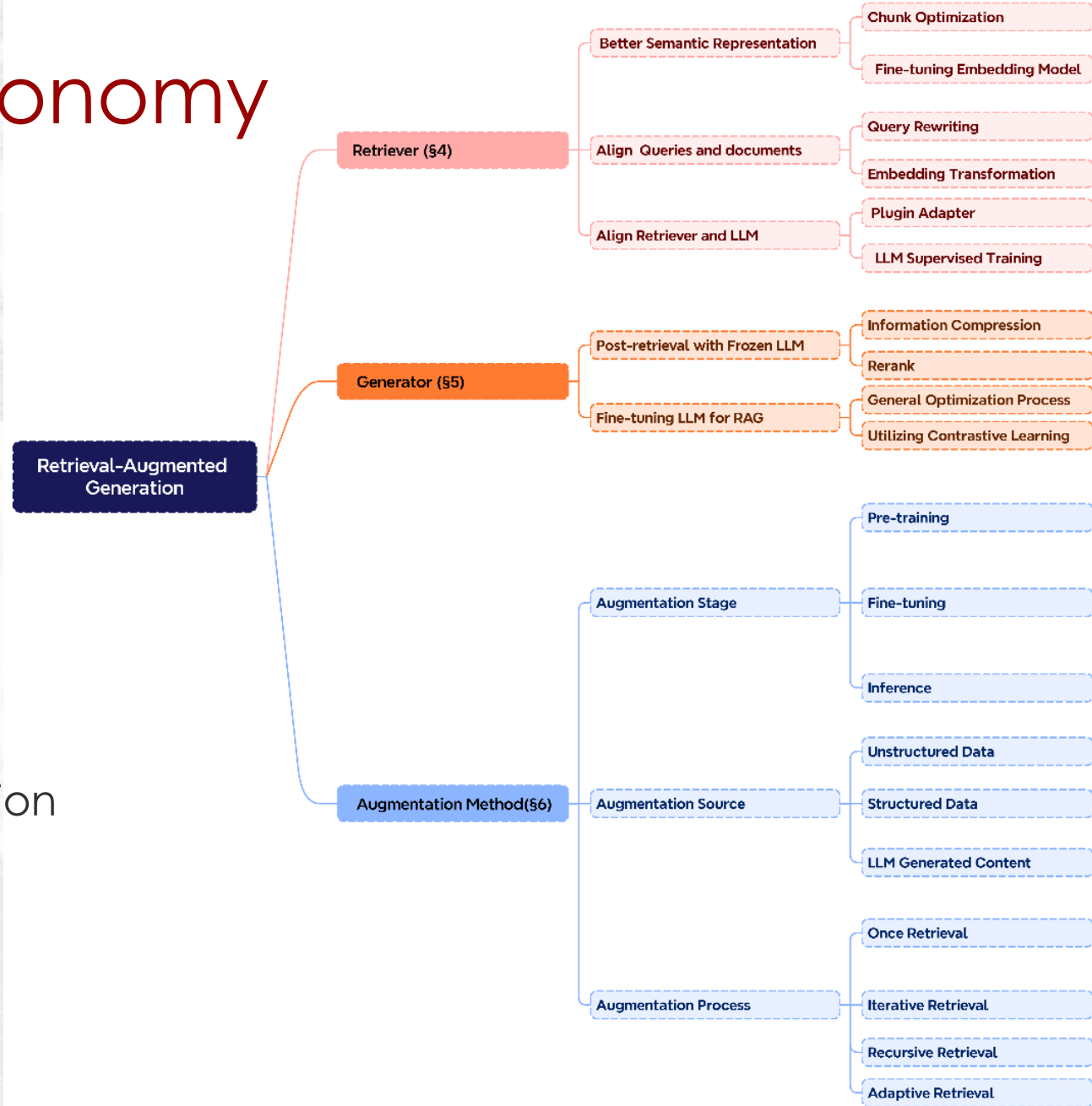
	Context Relevance	Faithfulness	Answer Relevance	Noise Robustness	Negative Rejection	Information Integration	Counterfactual Robustness
Accuracy	✓	✓	✓	✓	✓	✓	✓
EM					✓		
Recall	✓						
Precision	✓			✓			
R-Rate							✓
Cosine Similarity			✓				
Hit Rate	✓						
MRR	✓						
NDCG	✓						

Table 3: Summary of evaluation frameworks

Evaluation Framework	Evaluation Targets	Evaluation Aspects	Quantitative Metrics
RGB <sup>†</sup>	Retrieval Quality Generation Quality	Noise Robustness	Accuracy
		Negative Rejection	EM
		Information Integration	Accuracy
		Counterfactual Robustness	Accuracy
RECALL <sup>†</sup>	Generation Quality	Counterfactual Robustness	R-Rate (Reappearance Rate)
RAGAS <sup>‡</sup>	Retrieval Quality Generation Quality	Context Relevance	*
		Faithfulness	*
		Answer Relevance	Cosine Similarity
ARES <sup>‡</sup>	Retrieval Quality Generation Quality	Context Relevance	Accuracy
		Faithfulness	Accuracy
		Answer Relevance	Accuracy
TruLens <sup>‡</sup>	Retrieval Quality Generation Quality	Context Relevance	*
		Faithfulness	*
		Answer Relevance	*

# A RAG Taxonomy

- Research is active in different directions
  - Retrieval
  - Generation
  - Textual, Logical and Procedural Augmentation
- DBs or KG are often explored as information sources



# Retrieval-Augmented Generation

## Retriever (§4)

### Better Semantic Representation

#### Chunk Optimization

Small2big;Sliding-window;Abstract-Embedding;Metadata Filtering [Liu, 2023]

#### Fine-tuning Embedding Model

PROMPTAGATOR [Dai et al., 2022] ; BGE [BAAI, 2023]; LLM-Embedder [Zhang et al., 2023a] ; AngIE[Li and Li, 2023]

### Align Queries and documents

#### Query Rewriting

Query2Doc [Wang et al., 2023d]; RRR [Ma et al., 2023a]; STEP-BACKPROMPTING[Zheng et al., 2023]; HyDE [Gao et al., 2022]; TOC [Kim et al., 2023]

#### Embedding Transformation

SANTA [Li et al., 2023b]

### Align Retriever and LLM

#### Plugin Adapter

PKG [Luo et al., 2023]; RECOMP [Xu et al., 2023]; TokenFiltering [Berchansky et al., 2023]

#### LLM Supervised Training

AAR [Yu et al., 2023]; REPLUG [Shi et al., 2023] ; Atlas [Izacard et al., 2022] ; UPRISE [Cheng et al., 2023a]

## Generator (§5)

### Post-retrieval with Frozen LLM

#### Information Compression

PRCA [Yang et al., 2023a]; RECOMP [Xu et al., 2023] ; Filter-Reranker [Ma et al., 2023]

#### Rerank

Reranker [Brigger, 2023]; QLM [Zhuang et al., 2023]

### Fine-tuning LLM for RAG

#### General Optimization Process

Self-Mem [Cheng et al., 2023b]

#### Utilizing Contrastive Learning

SUGRE [Kang et al., 2023]; SANTA [Li et al., 2023b]

## Augmentation Method (§6)

### Augmentation Stage

#### Pre-training

RETRO [Borgeaud et al., 2022]; Atlas [Izacard et al., 2022]; REALM [Arora et al., 2023]; Toolformer [Schick et al., 2023]; COG [Lan et al., 2022]; RAVEN [Huang et al., 2023]; RETRO++ [Wang et al., 2023]; InstructRetro [Wang et al., 2023a]; TIGER [Rajput et al., 2023]

#### Fine-tuning

DPR [Karpukhin et al., 2020] ; UPRISE [Cheng et al., 2023a]; FID [Izacard and Grave, 2020]; RA-DIT [Lin et al., 2023]; Self-RAG[Asai et al., 2023]; SUGRE [Kang et al., 2023]; SANTA[Li et al., 2023b]; REPLUG [Shi et al., 2023]; AAR [Yu et al., 2023];

#### Inference

KNN-LM [Khandelwal et al., 2019]; DSP [Khattab et al., 2022]; KAR [Purwar and Sundar, 2023]; PRCA [Yang et al., 2023a]; IRCOT [Trivedi et al., 2022]; GenRead [Yu et al., 2022]; ICRALM [Ram et al., 2023]; PGRA [Guo et al., 2023]

### Augmentation Source

#### Unstructured Data

UPRISE [Cheng et al., 2023a]; CREA-ICL [Li et al., 2023a]; COG [Lan et al., 2022]

#### Structured Data

FABULA [Ranade and Joshi, 2023]; SUGRE [Kang et al., 2023]; KnowledGPT [Wang et al., 2023e]; GraphToolformer [Zhang, 2023]

#### LLM Generated Content

Self-Mem [Cheng et al., 2023b]; DSP [Khattab et al., 2022]; RECITE [Sun et al., 2022]; GenRead [Yu et al., 2022]; SKR [Wang et al., 2023f]

### Augmentation Process

#### Once Retrieval

REALM [Arora et al., 2023] ; RAG [Lewis et al., 2020]; UPRISE [Cheng et al., 2023a]; PKG [Luo et al., 2023]; LLM-R [Wang et al., 2023c] ; Atlas [Izacard et al., 2022]; REPLUG [Shi et al., 2023]; RECITE [Sun et al., 2022]

#### Iterative Retrieval

DSP [Khattab et al., 2022] ; Retrieve-Sample [Ren et al., 2023] ; ITER-RETGEN [Shao et al., 2023] ; ITRG [Feng et al., 2023]

#### Recursive Retrieval

IRCoT [Trivedi et al., 2022] ; ToC [Kim et al., 2023]

#### Adaptive Retrieval

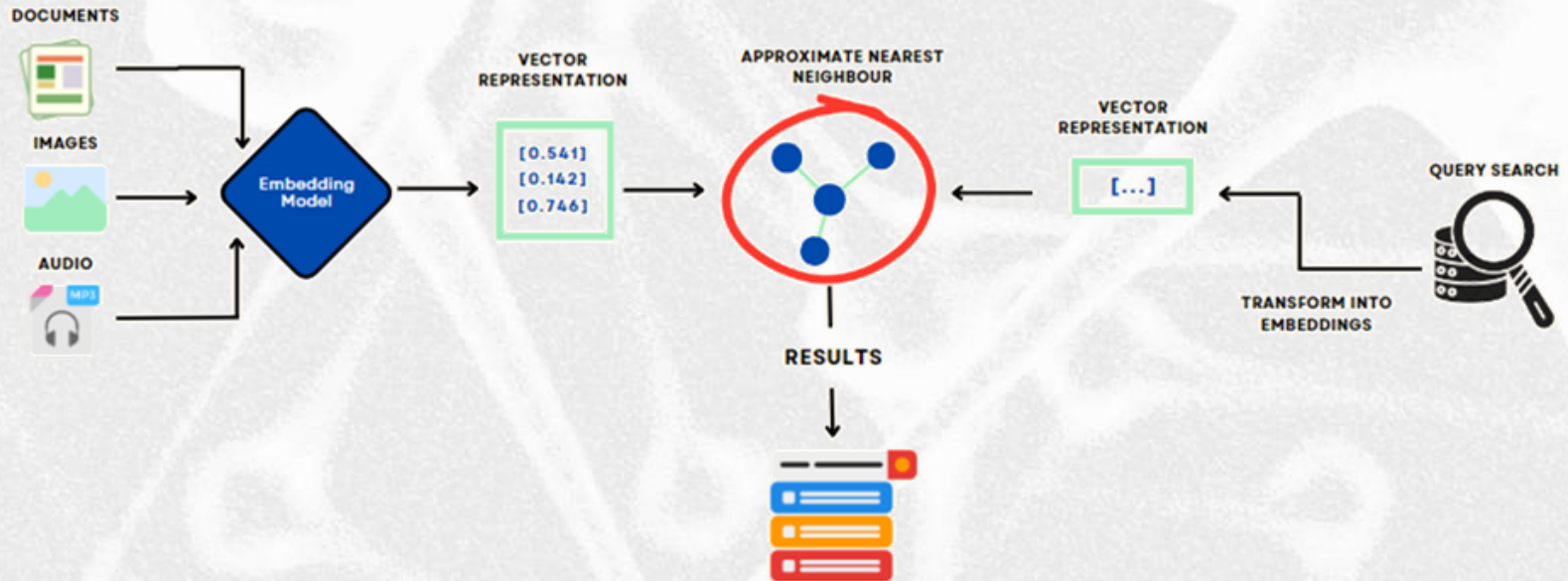
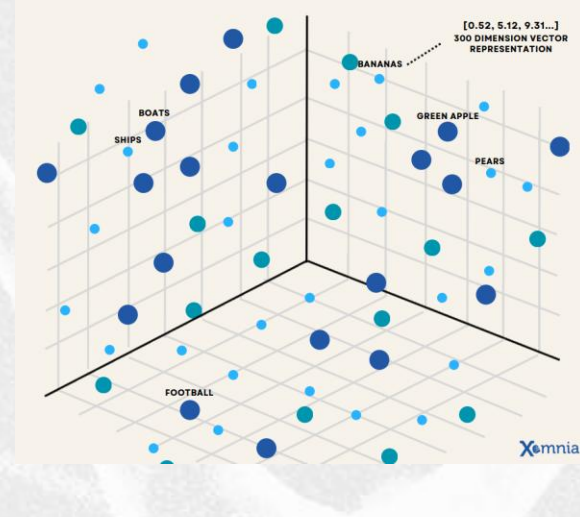
FLARE [Jiang et al., 2023] ; Self-RAG [Asai et al., 2023] ; RAVEN [Huang et al., 2023]



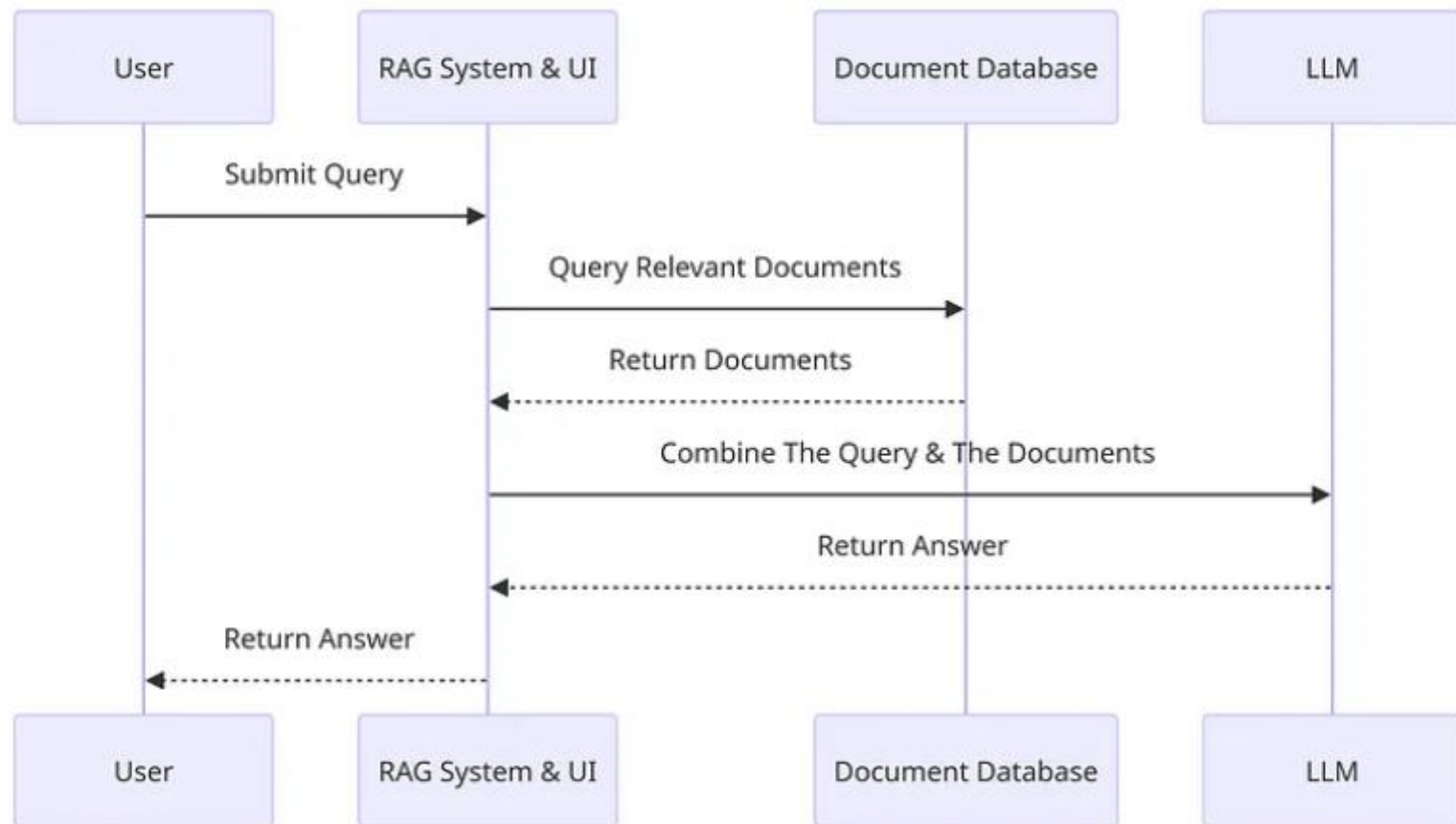
# Applications of RAGs

# Vector Databases

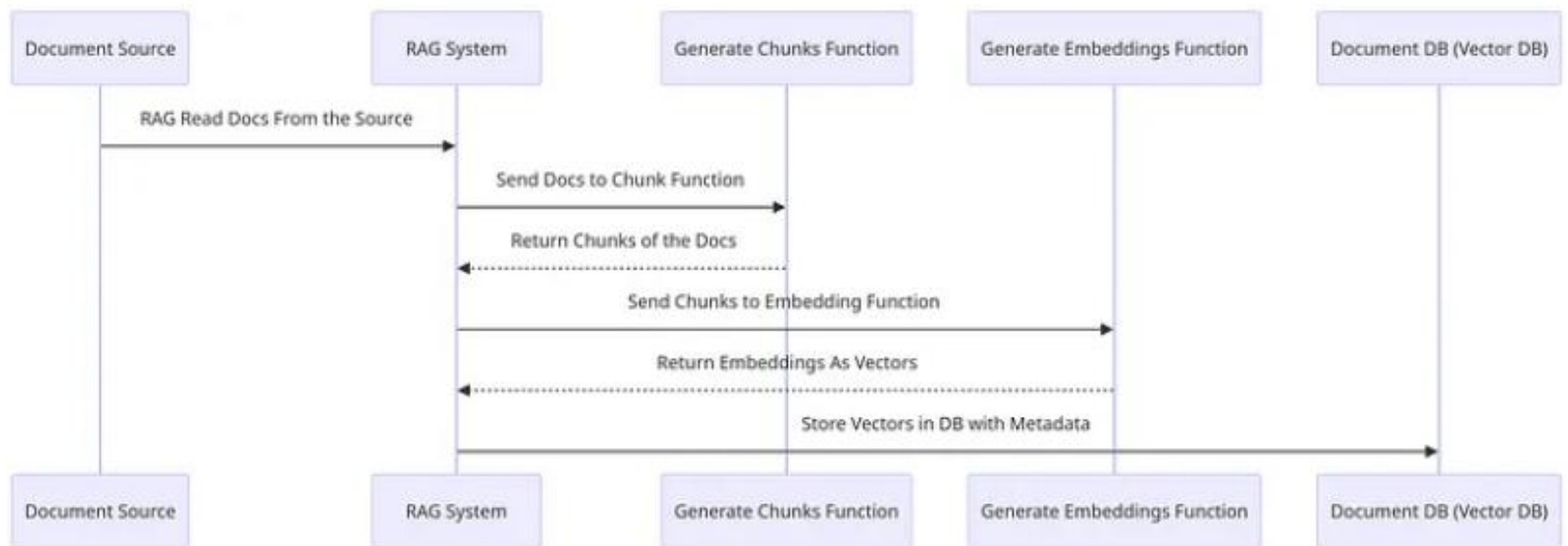
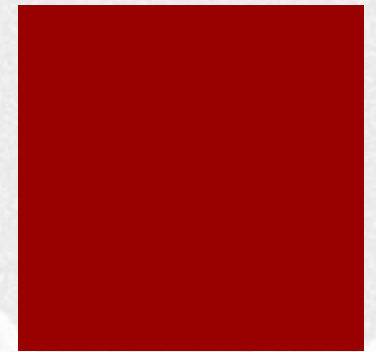
- A vector database is a type of database that stores and manages unstructured data, such as
  - texts, images, or audio,
- in vector embeddings (high-dimensional vectors) to make it easy to find and retrieve similar objects quickly.



# RAG: workflows



# RAG: data gathering

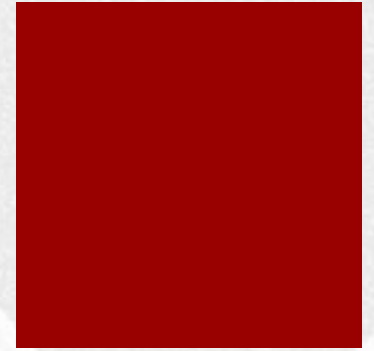


# RAG potential applications



- **Question Answering** where facts are derived from the retrieved texts that represent up-to-date information (in IR style)
- **Summarization**, where on-the-fly retrieval of supporting documents is carried out
- **DB query in NL**, as individual DB records can be seen as texts
- **KB retrieval** and **alignment to specific user' needs**
- ...

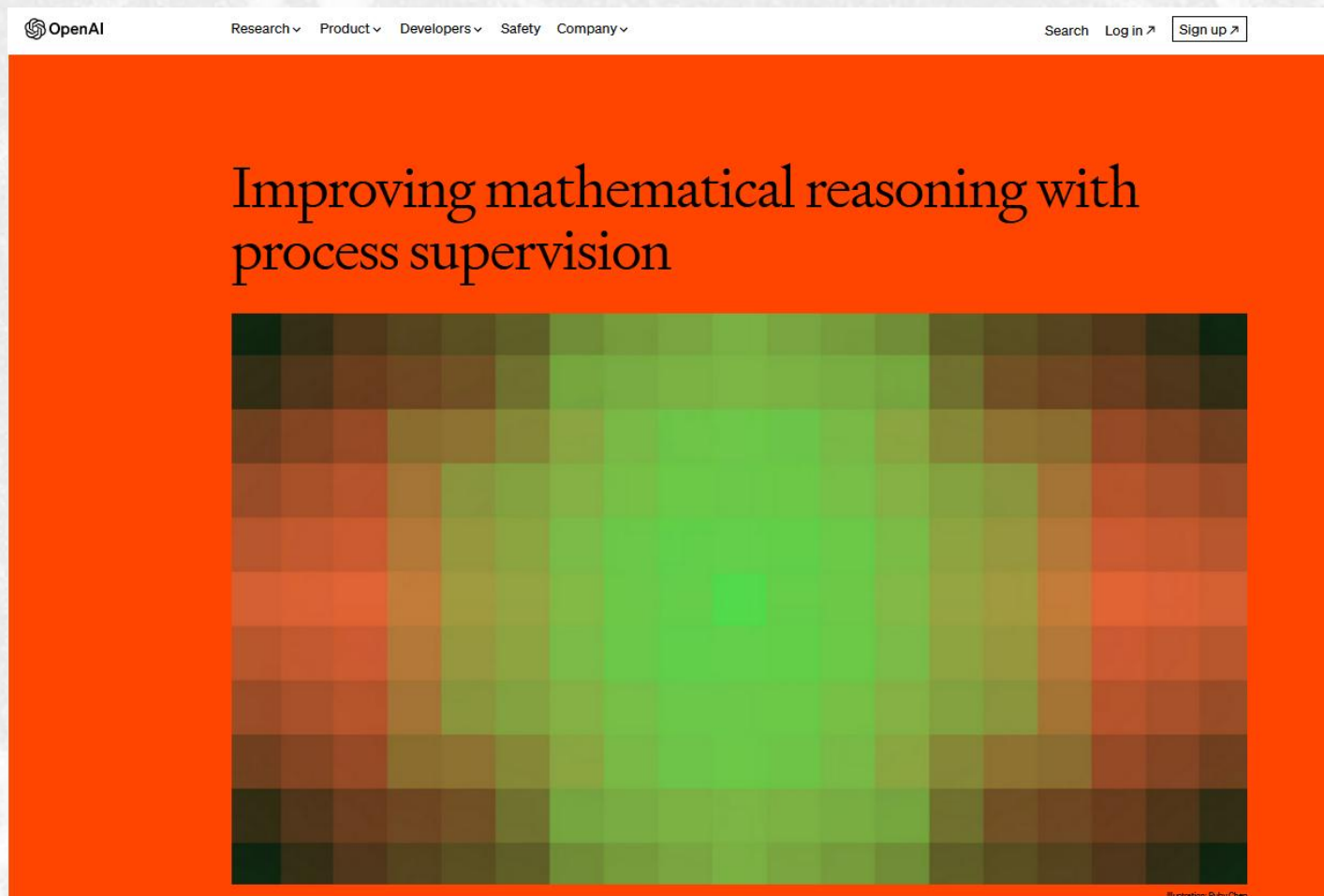
# RAG: business applications



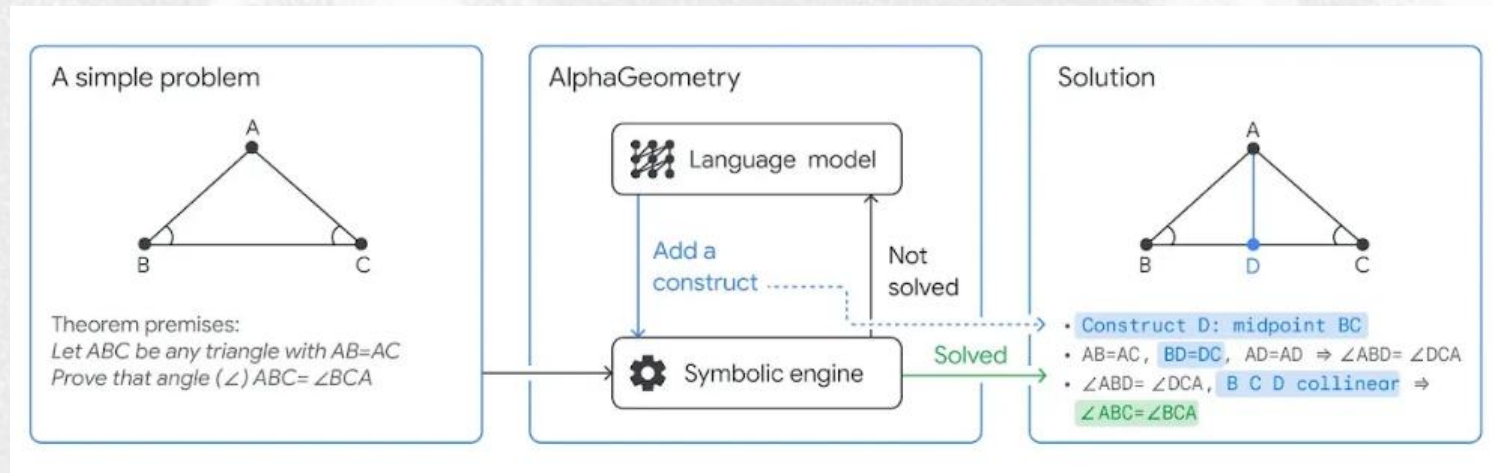
Practical applications of RAG include for example:

- **Customer support:** RAG can be used to build chatbots or AI assistants that provide personalized assistance across various questions and issues.
- **Content generation:** RAG enables the automation of content creation tasks, such as writing aids or content curation apps.
- **Education:** RAG can serve as a learning assistant, providing explanations and summaries of educational content.
- **Research:** RAG can assist researchers in obtaining relevant information and insights from large document collections.

# Future directions



# AlphaGeometry (Google DeepMind, Jan 2024)

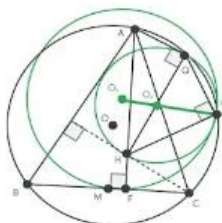


Trinh, Trieu H., Wu Yuhuai, Le Quoc V., He He, Luong Thang, Solving olympiad geometry without human demonstrations, Nature, 625, 2024.

# AlphaGeometry (Google DeepMind, Jan 2024)

## IMO 2015 P3

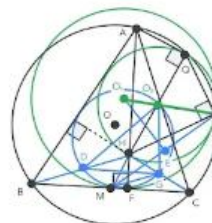
Let  $ABC$  be an acute triangle. Let  $(O)$  be its circumcircle,  $H$  its orthocenter, and  $F$  the foot of the altitude from  $A$ . Let  $M$  be the midpoint of  $BC$ . Let  $Q$  be the point on  $(O)$  such that  $QH \perp QA$  and let  $K$  be the point on  $(O)$  such that  $KH \perp KQ$ . Prove that the circumcircles  $(O_1)$  and  $(O_2)$  of triangles  $FKM$  and  $KQH$  are tangent to each other.



AlphaGeometry

## Solution

```
[...]
Construct D: midpoint BH [a]
[a], O2 midpoint HQ  $\Rightarrow$  BQ  $\parallel$  O2D [20]
[...]
Construct G: midpoint HC [b]
 $\angle GMD = \angle GO_2D \Rightarrow M, O_2, G, D$  cyclic [26]
[...]
[a], [b]  $\Rightarrow$  BC  $\parallel$  DG [30]
[...]
Construct E: midpoint MK [c]
[c]  $\Rightarrow \angle KFC = \angle KO_1E$  [104]
[...]
 $\angle FKO_1 = \angle FKO_2 \Rightarrow KO_1 \parallel KO_2$  [109]
[109]  $\Rightarrow O_1, O_2, K$  collinear  $\Rightarrow (O_1), (O_2)$  tangent
```



Problem 3 of the 2015 International Mathematics Olympiad (left) and a condensed version of AlphaGeometry's solution (right). The blue elements are added constructs. AlphaGeometry's solution has 109 logical steps.

Trinh, Trieu H., Wu Yuhuai, Le Quoc V., He He, Luong Thang, Solving olympiad geometry without human demonstrations, Nature, 625, 2024.

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- [ALoRA: Allocating Low-Rank Adaptation for Fine-tuning Large Language Models](#), liu et al., 2024
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  - (Lewis et al, 2020) [Retrieval-augmented generation for knowledge-intensive NLP tasks. Proceedings of NIPS, Advances in Neural Information Processing Systems, 33 \(2020\): 9459-9474.](#)
- RAG surveys & tutorial:
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  - "Towards LLM #8: Techniques of Prompt Engineering — Retrieval-Augmented Generation (Part 1)", LAKSHMI VENKATESH on [Luxananda - Medium.com](#)